

**"I think they are thinking that zero point something is less than zero":  
Investigating pre-service teachers' responses to mathematical tasks.**

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**Abstract**

It is generally accepted in education research that effective teaching of mathematics requires a combination of content knowledge and pedagogical content knowledge. What is less evident, however, is the relationship between these two types of knowledge and the impact this has on the learner. This paper reports on an investigation into pre-service teachers' mathematical content knowledge and their ability to interpret students' responses to tasks and make subsequent appropriate teaching decisions. Using a combination of quantitative and qualitative methods, the researchers tested aspects of the mathematical knowledge held by a cohort of final year pre-service primary teachers. From this group, a selected sample were given a number of hypothetical student work samples and asked to analyse the mathematical thinking and make suggestions as to appropriate teaching approaches. The results indicated that a lack of mathematical content knowledge did impact on the pre-service teachers' ability to both recognise and address students' mathematical errors. This study adds to the limited field of research by providing evidence that the types of knowledge required for effective teaching of mathematics are inextricably linked, with mathematical content knowledge being particularly influential. Furthermore, the results showed that the participants were often unaware that they lacked this knowledge and the impact this had on their teaching practice.

## Introduction

It is widely accepted in education research that effective teachers of mathematics have both knowledge of students' mathematical ideas and thinking as well as knowledge of mathematical content (Ball & Bass, 2000; Ball, Lubienski & Mewborn, 2001; Hill, Ball & Schilling, 2008). The seminal work of Shulman (1986) and his colleagues during the 1980s highlighted the importance of these aspects of teacher knowledge and distinguished between the terms content knowledge and pedagogical content knowledge. Content knowledge is concerned with expertise in the particular discipline being taught and in this paper such knowledge will be referred to as mathematical content knowledge (MCK). Pedagogical content knowledge (PCK), encompasses all that is needed to teach a particular subject or topic that makes it comprehensible to others including an understanding of what makes the learning of a particular topic easy or difficult and the common conceptions and misconceptions that students may bring with them.

To be able to provide sound explanations of mathematical ideas demand that the teacher has rich and correct connections across each of the domains of mathematics curriculum they are required to teach (Ball, 2000; Ma, 1999). Research suggests however, that many pre-service primary school teachers exhibit similar difficulties with some foundational mathematical skills and concepts as the students they are required to teach (Ball, 2000; Ryan & Williams, 2007; Stacey et al., 2001; Tirosh, 1999).

This paper reports on the preliminary findings from a study designed to investigate in what ways if any, pre-service teachers' MCK impacts on their ability to teach mathematics across several domains relevant to the primary school mathematics curriculum. The domains in the study included aspects of rational number, measurement, early algebraic thinking, statistics and probability, with the domain of rational number, and specifically decimal numeration, being the focus of this paper. Although there has been extensive research into students' understanding of key mathematical ideas (Asquith, Stephens & Alibali, 2006; Ball, 2000; Fischbein & Schnarch, 1997; Graeber, 1999; Stacey et al., 2001), there is, however, limited research into the impact that pre-service teachers' MCK has on their ability to interpret students' approaches to mathematical tasks and their ability to determine appropriate teaching strategies to address student errors and misconceptions. This paper will attempt to add to the limited knowledge in this area through answering the following research questions:

1. What is the nature of pre-service primary school teachers' MCK relevant to primary school mathematics (specifically in the domain of decimals)?
2. To what extent can pre-service primary teachers identify students' errors in mathematical tasks?

3. To what extent can pre-service primary teachers identify appropriate teaching strategies to address students' mathematical misconceptions?

Review of the literature

### **Mathematical Knowledge for Teaching**

A strong knowledge of mathematics is necessary for effective mathematics teaching, and teachers who lack such knowledge may be limited in their ability to help students develop conceptual and relational understanding of mathematics (Skemp, 1978). In his seminal work, Skemp distinguished between two kinds of understanding in mathematics; instrumental and relational. Relational understanding is concerned with the underlying principles of a particular mathematical idea whereas instrumental understanding involves following rote learnt rules and procedures, that is "rules without reason" (Skemp, 1978, p. 9). Whilst instrumental understanding may afford students the opportunity to get the correct answers to certain mathematical items in a very specific context, relational understanding involves the learner building a conceptual structure to solve mathematical problems in a range of contexts.

In her comparative study of Chinese and American elementary (primary) teachers, Ma (1999) argued that effective mathematics teaching is based on profound understanding of fundamental mathematics (PUFM). According to Ma, teachers with PUFM possess a thorough and well connected understanding of primary mathematics which includes understanding how concepts and procedures are related to each other and to more advanced mathematical ideas. Ma's study contributed to the body of research that indicate that many pre-service primary teachers have weak understanding of many of the mathematics skills and concepts that they are required to teach. Similarly, the work of Ball and her colleagues, emphasised the importance of pre-service teachers developing a richly connected understanding of the relationships between mathematical concepts, procedures and topics (Ball, 2000; Ball & Bass, 2000).

The transformation of MCK into PCK should be an important focus in teacher education (Ryan & Williams, 2007), and teachers need to make their own understanding of mathematical content explicit in order to make this content comprehensible to others. A number of frameworks based on the work of Shulman (1978) for identifying and describing pre-service teachers' and teachers' MCK and PCK have been developed by mathematics education researchers (e.g., Hill et al., 2008; Chick, Baker, Pham & Cheng, 2006; Rowland, Huckstep & Thwaites, 2003). The framework for mathematical knowledge for teaching (MKT) developed by Hill and colleagues encompasses PCK and subject matter knowledge (SMK) (see Figure 1) and is a refinement of Shulman's original categorization of subject matter knowledge and PCK. Hill et al.'s domain of SMK is further

delineated into common content knowledge (CCK), specialised content knowledge (SCK), and knowledge at the mathematical horizon. CCK is defined as the knowledge used in teaching any discipline that involves the use of mathematics whereas SCK involves being able to represent mathematical ideas accurately and interpret unusual solutions to mathematical problems (Hill et al., 2008). Knowledge at the mathematical horizon is defined as an awareness of how mathematical topics are connected throughout the mathematics curriculum (Ball, Thames, & Phelps, 2008). Similarly, PCK is further divided into knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of the curriculum. KCS and KCT are separate from each other as subsets of PCK (see Figure 1). Whilst KCS focuses on teachers' understanding of how students learn particular content, KCT is concerned with how best to build on students' mathematical thinking and how to address student errors (Hill et al., 2008). Aspects of this framework which are particularly relevant to this paper and which informed the theoretical framework include PCK, KCS and KCT and were used to define the three knowledge domains as identified in Table 2.

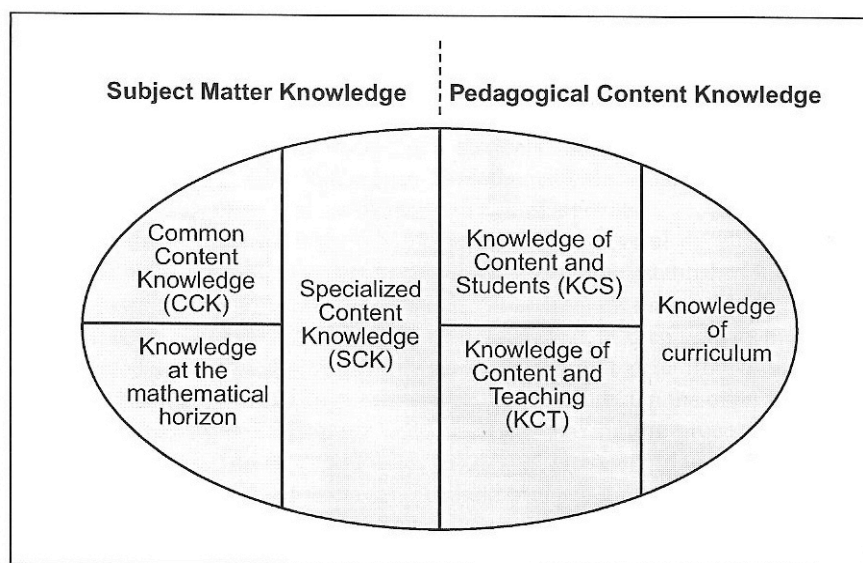


Figure 1. Domain map for mathematical knowledge for teaching.

### The Foundational Ideas of Specific Mathematical Knowledge

Shulman's conceptualisation of PCK contributed greatly to current mathematics education research into what teachers need to know about students' learning (Even, 2008). A key aspect of PCK includes students' typical pre-conceptions or misconceptions and the strategies that teachers use to overcome misconceptions which align with Ball and colleagues KCS and KCT respectively. A misconception may be defined as the misapplication of a mathematical rule or an over or under generalisation of a mathematical idea. For example, a common misconception in relation to

decimal numeration particularly in younger students, involves inappropriate whole number thinking whereby a student may suggest that 0.284 is larger than 0.35 since 284 is larger than 35. Research suggests that students construct their knowledge of mathematical concepts and ideas in ways which “differ from what is assumed by the professional community” (Even & Tirosh, 2008, p. 203).

This paper specifically focuses on participants’ responses to decimal tasks, an area which has been recognised as a source of teaching and learning difficulties (Stacey et al., 2001; Steinle & Stacey, 1998). Stacey et al. (2001) found, for example, that at least one in five of the pre-service primary teachers who participated in their study demonstrated inadequate knowledge of decimal numeration with the related risk of transferring these misconceptions to students.

An important pattern of consistent errors in decimal numeration relate to what are termed ‘shorter-is-larger’ misconceptions (Stacey et al., 2001; Steinle & Stacey, 1998). Studies have found that the ‘shorter-is-larger’ misconceptions persist throughout primary and secondary school with around 7% of Year ten students demonstrating one or more decimal misconceptions of this kind with the prediction that this would continue into adulthood (Stacey et al., 2001; Steinle & Stacey, 1998). The ‘shorter-is-larger’ category can be further divided into reciprocal, negative and denominator focused thinking (Stacey et al., 2001). Reciprocal thinking refers to consistent errors that associate decimals with reciprocals leading to inappropriate conclusions such as 0.3 is larger than 0.4 since  $\frac{1}{3}$  is larger than  $\frac{1}{4}$ . Negative thinkers may associate decimals with negative numbers; for example, they may suggest that 0.6 is smaller than zero in the same way that negative six is smaller than zero. Denominator focussed thinkers tend to use place value column names to decide on the relative size of decimals, and may, for example, perceive 0.3 as being larger than 0.426 based on the reasoning that any number of tenths must be greater than any number of thousandths (Stacey et al., 2001).

Another relevant finding from the work of Stacey and her colleagues was the fact that most pre-service teachers who participated in the study were aware of the ‘longer-is-larger’ misconception in younger students but had little awareness of the ‘shorter-is-larger’ misconception (Stacey et al., 2001). Whilst findings indicated that the pre-service teachers were able to identify features that make decimal comparison difficult, they were less able to explain why they were difficult (Stacey et al., 2001).

## Methodology

There were two phases to the study. The first phase of the study involved the completion of a 15 item survey instrument consisting of nine multiple choice and six short answer questions relating to

mathematical content relevant to the Australian primary school curriculum. The multiple-choice items were selected from a sample ACER Teacher Educational Mathematics Test (TEMT) designed to test the mathematical attainment of beginning pre-service teachers and to uncover errors due to misconceptions. The short answer test items were adapted from other studies involving research into the MCK held by pre-service and in-service primary school teachers (Ma, 1999; Ryan & Williams, 2007; Stacey, et al., 2001). Twenty final year B. Ed pre-service teachers volunteered to take part in this first phase. At the end of the survey, the participants could indicate an interest in participating in the second phase of the study and seven of them agreed to do this.

The second phase consisted of an interview which was structured around four key questions or instructions relating to six student work samples. The work samples were constructed by the researchers, and adapted from previous studies on students' mathematical thinking in various topic areas (Ma, 1999; Ryan & Williams, 2007; Stephens, 2006). The mathematical concepts that underpinned the tasks directly reflected those on the survey instrument. Table 1 shows the items discussed in this paper from the survey instrument, the corresponding work samples and the primary questions asked. The interviews took approximately one hour and were audio-taped. They were then transcribed and analysed using an adaptation of the framework for conceptualising MKT based on the work of Hill, et al. (2008). Whilst other frameworks for PCK have been developed (e.g., Chick, et al., 2006; Rowland, et al., 2003), the authors considered the domain map of mathematical knowledge for teaching developed by Hill and colleagues to be the most useful in devising the three categories of 'knowledge of content', 'knowledge of learning' and 'knowledge of teaching' (see Table 2). Each participant's response in relation to each of the interview tasks was assigned a rating between 0-2 based on the rating descriptors for the three domains of knowledge. To assess the appropriateness and reliability of the rating scheme, a second coder read and rated the interview transcript for each participant. The interview questions were structured to answer the second and third research questions.

*Table 1: Survey item, corresponding work samples and primary interview questions*

Survey item	Corresponding work sample	Questions asked
Q. 10 Circle the largest number in the following pairs: 0.75 0.8 0.426 0.3 3.72 3.073 8.245 8.24563 0.3 0.4	Work sample 1 A student completes the following decimal items as follows: Circle the largest number in each of the following pairs 0.4 0.87 4.4502 4.45	Please indicate whether you think the student is correct or incorrect for each of the items. What does this tell you about the student's thinking of decimals? Can you give an example

0	0.6	<p>0.12 0.3456</p> <p>0.37 0.2369</p> <p>Place the following numbers in order from smallest to largest 3.303, 0, 3.03, 3.33, 3.033, 0.3, 0.3333</p> <p>0.3, 0.3333, 0, 3.033, 3.03, 3.303, 3.33</p>	<p>of a decimal comparison that they would get wrong based on that thinking?</p> <p>What would you do to assist the child to overcome these problems?</p>
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The coded responses for each of the seven participants were compared with their individual responses to the survey items administered in the first phase of the study as shown in Table 3. An ideal response (i.e., rating 2) in the 'knowledge of content' (MCK) category for the interview would include correct identification of the largest decimal in each pair for the circling task and correct sequencing (ascending order) of the string of decimals in the ordering task. An ideal response in the 'knowledge of learning' category would demonstrate awareness of the shorter is larger misconception in the circling task and negative thinking in the ordering task. Furthermore, a high quality response to the ordering task may also comment on, but not necessarily explain, the mix of misconceptions including contradictions to the previously identified 'shorter is larger' pattern. In the 'knowledge of teaching' category, an ideal suggested teaching strategy to assist with addressing the identified misconceptions may emphasise the additive structure of the base ten numeration system (Steinle, 2004). For example, such a strategy may employ concrete materials such as Linear Arithmetic Blocks (LAB) to compare the length of 0.87 which is 8 tenths plus 7 hundredths with 0.4 which is 4 tenths.

Of particular interest was any relationship between the participants' MCK as demonstrated in the survey and the nature of their responses in relation to their interpretation of the student work samples and strategies for addressing student mathematical misconceptions. The results obtained from the seven participants' responses to the decimal work sample are discussed in the following section.

*Table 2. Rating descriptors for each knowledge domain*

Knowledge domain	0	Rating descriptors 1	2
Knowledge of content (MCK)	Could not solve problem correctly	Solved problem but with limited fluency and/or	Solved problem, demonstrating procedural



		confidence	proficiency and/or relational understanding
Knowledge of learning (MCK/PCK/KCS)	Did not identify error in student's thinking	Identifies error but no or limited explanation given	Identifies error in thinking and provides appropriate explanation
Knowledge of teaching (PCK/KCT)	No teaching strategy identified	Identifies teaching strategy with no or limited reference to instruction and/or focus on instrumental understanding only	Identifies appropriate strategy or approach for developing students' relational understanding of mathematical concept.

## Results and Discussion

Table 3 summarises the participants' responses to the decimal comparison items on the survey instrument and the student work sample. The incorrect responses received on the survey instrument are highlighted in bold font. The participants' ratings of either 0, 1, 2 are included for each of the three categories as determined by their answers in the interview. N/a has been used to denote that the participant did not identify the student's answer as incorrect and therefore did not progress any further in the interview.

### Knowledge of content

Within the knowledge of content category, only three participants, Mia, Courtney and Jacky correctly answered the six decimal comparison questions on the survey instrument. Courtney also correctly identified the errors in the interview questions as evidenced by the two '2' ratings. Both Ann and Janet's answers to question 4 show evidence of 'shorter is larger' thinking (Stacey et al., 2001; Steinle & Stacey, 1998), which is also indicative of Sarah and Janet's answer to question 2. Both Ann and Janet incorrectly circled zero as being larger than 0.6, which suggests a pattern of negative thinking. Other examples of this thinking occurred during the interview, with four participants, including Ann and Janet, implying that zero was larger than all positive decimals less than one. Whilst Mia and Larissa correctly identified 0.6 as greater than 0 in the survey, they overlooked the incorrect placement of zero in the decimal sequence during the interview. It is however, worth noting that the choice of number examples in the sequence may have made the ordering especially challenging. Mia, for example, noted that: "this one is obviously confusing with the numbers being used with 3.33, 3.033 and zeros everywhere". Three of the seven participants, Sarah, Courtney and Jacky, however, correctly placed the sequence of numbers in order of size.



Table 3: Summary of participants' responses in relation to decimal numeration

Participant	Knowledge of Content							Knowledge of Learning	Knowledge of Teaching	
	MCK instrument				Interview					
	Circle largest 0.7	Circle largest 0.426	Circle largest 3.723	Circle largest 8.24563	Circle largest 0.3	Circle largest 0.4	Circle largest 0.6	Order sequence of decimals	Identification of error	Identification of teaching strategy
Ann	0.8	0.426	3.72	<b>8.245</b>	0.4	<b>0</b>	1	0	1	1
Mia	0.8	0.426	3.72	8.24563	0.4	0.6	2	1	1	1
Sarah	0.8	<b>0.3</b>	3.72	8.24563	0.4	0.6	2	2	1	1
Janet	0.8	<b>0.3</b>	3.72	<b>8.245</b>	0.4	<b>0</b>	0	0	n/a	n/a
Larissa	<b>0.7</b>	0.426	3.72	8.24563	<b>0.3</b>	0.6	1	1	0	n/a
Courtney	0.8	0.426	3.72	8.24563	0.4	0.6	2	2	1	1
Jacky	0.8	0.426	3.72	8.24563	0.4	0.6	1	2	1	1

Larissa's answers in the survey instrument show that she incorrectly identified 0.7 as being larger than 0.8, and 0.3 as being larger than 0.4. Interestingly, she did not show 'shorter is larger' thinking when comparing decimals with different numbers of digits. Instead it seems that the source of Larissa's error may have been related to reciprocal thinking (Stacey et al., 2001). During the interview, she provided the following explanation as to why 0.2369 was larger than 0.37:

37 hundredths and 23169 hundredths um if you just break it up to 23 hundredths it is going to be less parts so it is going to be bigger than 37 hundredths.

In other words she rounded 0.23169 to 0.23 and compared the two equal length decimals, 0.37 and 0.23 and incorrectly concluded that 0.23 is larger than 0.37 in the same way that  $1/23$  is larger than  $1/37$ . Further evidence of her reciprocal thinking was demonstrated in the following response:

Um ... the student is right with that one the first one. Four tenths is bigger than eight tenths because um eight tenths is smaller because it has more pieces.

### **Knowledge of learning**

Ratings in this category included both the circling and the ordering tasks. After being asked to identify which items were incorrect in the work sample, the participants were then asked to elaborate on what the answers might reveal about students' thinking. Participants' answers were rated using the criteria in Table 2. As Table 3 shows, none of the participants were able to identify both the shorter is larger thinking in the circling task and the negative thinking associated with the ordering task. Jacky, Sarah and Courtney were the only participants to recognise the negative thinking in the ordering task. Although none of the three participants used the term "negative thinking" in their responses they did recognise that the sequence pivoted at zero instead of one. "I think that they think that anything that is zero point something is going to be less than zero rather than more than zero and less than one"(Sarah).

Some of the participants were able to provide some explanation for the students' errors in the circling task, but did not identify errors in the ordering task, and therefore received a rating of '1' for their responses. For example Mia, attempted to explain the shorter is larger misconception but her reasoning was ambiguous and difficult to follow, particularly in relation to her suggestion about the relative distance of the decimals from one.

They might not be aware of anything after the tenths because they've circled 4.45 rather than 4.4502 which is actually bigger but they've ignored those numbers altogether or maybe they think it means it's less or something. Maybe they are thinking that it has to be closer to one. Like with 0.4 and 0.87 they might think of 0.4 as being closer to a whole number because 4 is closer to one than 87.

Janet and Larissa were rated either n/a or 0 as Janet did not progress beyond examining the work sample and Larissa did not demonstrate an awareness of any error patterns consistent with the work sample. Whilst Ann expressed uncertainty in relation to her own knowledge of some aspects of decimal numeration she was able to articulate that comparing decimals of unequal length appeared to be a source of difficulty for the student who completed the work sample:

Well I think they would be ok with if you went [writes 0.45 and 0.46]. If you kept them the same tenths or hundredths. If you add digits on the end ... [writes 0.147 and 0.1472] ... I think anything that's got the variance with the decimal place you know the tenths and hundredths afterwards might be confusing.

Courtney, Sarah and Jacky were able to provide feasible examples of other decimal pairs that the student would be likely to get correct and incorrect, based on the evidence gleaned from the work

sample. For example Courtney selected 0.12 and 0.123 and suggested that “0.12 would be chosen as biggest going on what they are thinking about in the questions before.” Similarly, Sarah suggested that the student would most likely “believe that 0.5 is larger than 0.507 and would correctly state that 0.8 is greater than 0.721”.

It is notable that whilst some participants were able to devise similar decimal comparison tasks and accurately predict the student’s response to them, no one provided ideal responses in the knowledge of learning category. That is, none of the participants were able to identify and explain both the “shorter is larger” misconception in the circling task and the “negative thinking” in the ordering task. Although we recognise that it is possible to provide an ideal response without specifically using the terms ‘shorter is larger’ or ‘negative thinking’ none of the participants used this terminology in their responses.

### **Knowledge of teaching**

Table 3 shows that five participants received a rating of ‘1’ for their identification of an appropriate teaching strategy within the domain of ‘knowledge of teaching’. Janet was not asked to provide a suggestion as she did not identify the error in the work sample, while Larissa also received an ‘n/a’ for her response as she became confused with explaining the thinking behind the student’s error, and admitted to “getting myself all flustered”.

Common responses which identified a teaching strategy, but were limited in terms of detail about instruction, tended to focus on the use of place value charts and number lines. The participants’ responses provided little detail of how the strategy might be implemented, however, to enhance the student’s relational understanding of decimal numeration as the following suggestion from Courtney illustrates:

Going back to teaching that it is different to teaching ones, tens on the other side of the decimal point that it goes tenths and hundredths and so forth... I think maybe using visuals with the number line and things like that actually doing it themselves umm and getting peer feedback umm to help them understand the misconceptions they are making.

Sarah and Jacky suggested the use of a number line to address the student’s misconception in relation to negative thinking about decimals being less than one. Their strategies, however, tended to rely on showing the student the correct order rather than developing the student’s conceptual understanding of place value, as Jacky’s suggestion shows:

I'd use things like number lines with them and I'd get them ordering like ordering these types of numbers on a number line one that had some numbers on there and getting them to fill in the gaps...and showing them that if the number is going to go between 0.5 and 0.7 so if you've got 0.6 it is closer to the 1 it's going to be bigger than you know like 0.3.

Sarah also suggested the use of a number line to address the negative thinking evident in the ordering task. "I'd probably get out a number line and say this is where zero is and this is where 1 is and everything lies between the zero and the one" (Sarah). Although the number line may be useful in discussing decimal density (Steinle, 2004), Sarah's suggestion does not address the base ten structure of decimals.

Despite demonstrating tentative MCK of certain aspects of decimal numeration, Ann did attempt to articulate a teaching strategy that explicitly focused on developing the student's understanding when comparing decimals of unequal length. The following explanation refers to the comparison between 0.147 and 0.1472:

I think it is the decimal place that is causing the problems...so lots of work you know with the decimal place...where you actually put the counters on the pole bits they are actually made with the decimal point marked...they are blocks like planks of wood and you had little discs to put on them...you've got the 147 and then you have the 1472 (Ann drew a model of the resource and pointed to the 10000ths "pole"). You've got your two here but this might be completely wrong because you've got more counters here... does that give you an indication that it is wrong?

The 'poles' Ann referred to were Linear Arithmetic Blocks (LAB) which the pre-service teachers had been exposed to throughout their mathematical units. Her explanation, however, shows that she was not confident with explaining how she would actually use them to further students' understanding and later expressed concern about her ability to use the resource effectively:

I'm not sure I know enough to explain how to work it out for themselves. I certainly wouldn't be confident that I would be leading them up the right path ... I mean obviously I'd use concrete materials but I would have to research if I was using the right ones.

Whilst Ann could recall some characteristics of LAB, she did not explain how the resource provided a physical representation of the base ten structure of the decimal system by attending to both the

varying sizes and number of components according to the different place values. In other words she had not yet made the link between her own knowledge of the LAB and how it can be used to develop students' conceptual understanding of decimal numeration. Teaching strategies identified by other participants also tended to focus on instrumental understanding in relation to decimal numeration, and little detail was provided about instruction.

As was the case in the knowledge of learning category, no ideal responses were provided by the participants in the knowledge of teaching category. More specifically none of the participants emphasised the additive structure of the base ten numeration system.

### Conclusions

The participants' responses to the test items and the interview task showed that some of them exhibited the same misconceptions about decimals as identified in the literature. In particular, there was evidence of 'shorter-is-larger' and 'negative thinking' and this in turn impacted upon the participants' abilities to identify these misconceptions in students and therefore attempt to address this thinking with appropriate teaching strategies. Janet, for example, provided incorrect answers for three of the items in the test instrument, was subsequently unable to identify the errors in the work sample, and therefore would not have been able to help the student with developing an understanding of the comparative size of decimal numbers.

No ideal responses were received by the participants in either the 'knowledge of learning' or 'knowledge of teaching' categories. Findings also suggest that even participants who showed greater understanding of decimals still tended to provide a limited explanation as to the student's thinking and perhaps not surprisingly, this then restricted their ability to provide an appropriate approach or teaching strategy to develop conceptual understanding in students. Teaching strategies suggested tended to focus on either procedural approaches, such as Jacky's explanation of the use of the number line, or on using concrete materials, with little explanation of how they would be used to assist with addressing the error. Ma, (1999) found that the depth and quality of students' mathematical thinking depends on the teacher's understanding of mathematics, regardless of whether the teacher encourages such practices such as group work and the use of manipulatives. The implications from this study point to the need for pre-service teachers to be aware of the importance of having strong MCK and the influence that this has on their PCK. Moreover, evidence from this small study tends to suggest that the nature of the MCK held by the pre-service teachers is of particular importance. That is having an understanding of conceptually powerful mathematical ideas which include demonstrating mathematical connections and fluency of both concepts and procedures is necessary for effective teaching. Furthermore, it is of particular

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concern that these pre-service teachers were in their final year of study and therefore unlikely to address their own misconceptions before teaching others. It is hoped that this study contributes further to investigating the link between MCK and PCK and identifying the aspects of pre-service teachers' knowledge that needs to be focused on in the delivery of education courses.

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