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Endogenous product scope: Market interlacing and aggregate business cycle dynamics

Oscar Pavlov [a,b], Mark Weder [c,b,*]

[a] Tasmanian School of Business and Economics, University of Tasmania, Australia
[b] Centre for Applied Macroeconomic Analysis, Australian National University, Australia
[c] Department of Economics and Business Economics, Aarhus University, 8210 Aarhus V, Denmark

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ABSTRACT

This paper examines a market interlacing industry configuration in general equilibrium with multi-product firms. In contrast to previous studies which utilize market segmentation, firms produce multiple products even in the complete absence of the love of variety. Product scopes are procyclical and entry and exit of firms generates an endogenous amplification mechanism. When simulated by shocks derived from the efficiency and labor wedges, the model replicates the changes in dynamics between the pre- and post 1983 periods, and explains the hours-productivity puzzle.

1. Introduction

A significant portion of firms are multi-product producers. Bernard et al. (2010) report that close to half of U.S. manufacturing firms produce in multiple 5-digit SIC industries. These firms account for well over 80 percent of total sales. Furthermore, in excess of 90 percent of product creation and destruction occurs within these firms (i.e. as firms adjust their product scopes) as documented by Broda and Weinstein (2010). Additionally, Guo (2021) presents evidence that the scope of products that firms bring to the market is significantly procyclical. These observations suggest an important role of multi-product firms in shaping the dynamics of aggregate output.

Why do multi-product firms emerge? As emphasized by Bailey and Friedlaender (1982), the firms' existence is enmeshed in realizing cost advantages arising from economies of scope or scale. An alternative motivation, also based on a form of increasing returns to scale, is a *love of variety* effect coming from, say, a Dixit and Stiglitz (1977) demand system. Along these lines, Feenstra and Ma (2009), Minniti and Turino (2013) and Pavlov and Weder (2017), introduce multi-product firms into a general equilibrium framework and discuss their effects on aggregate economies. What these works share is the specification of the CES-aggregator function of goods. All three follow Brander and Eaton's (1984) *market segmentation* platform in which each nest of goods corresponds to a multi-product firm's output of close substitutes.[1] That is, the aggregator pulls together varieties from a single firm. Brander and Eaton (1984) also discuss situations where *market interlacing*, instead of segmentation, can arise as an equilibrium outcome.[2] In this other platform, the objects in a nest are similar goods (say, electric cars) that are brought to the market by different multi-product firms. While not claiming that this market interlacing alternative is necessarily a more appropriate case, here we take the route in which similar goods from different firms form close substitutes. One of the key takeaways, as we will show, will be that under

* Corresponding author at: Department of Economics and Business Economics, Aarhus University, 8210 Aarhus V, Denmark.
E-mail addresses: oscar.pavlov@utas.edu.au (O. Pavlov), mweder@econ.au.dk (M. Weder).

[1] See also Allanson and Montagna (2005) and Doraszelski and Draganska (2006).

[2] Chen and Chen (2014) find that market interlacing can emerge instead of segmentation when firm asymmetries are considered.

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this formulation increasing returns to scale in the aggregation of products in the form of variety effects are no longer required for multi-product firms to exist.[3]

In the market segmentation arrangement, final output is a combination of product nests with each nest consisting of varieties produced by the same firm. Due to the *love for variety*, the value of this nest to the consumer in terms of an effective consumption basket is higher with the number of different varieties. This form of increasing returns gives firms an incentive to produce multiple products i.e. firms are able to increase their market share by providing more varieties with the resulting revenue being able to cover the costs of developing or marketing the additional product. However, without the love of variety, the consumer does not value product differentiation and introducing new products only leads to a loss of profits. That is, a new product is only going to reduce the market share of the firm's other products and it is then optimal to produce a single product. Pavlov and Weder (2017) show exactly this. Market interlacing packaging, on the other hand, nests different firms' similar goods i.e. a product category. A single firm deciding to produce a new variety, in a product category that no other firm is producing in, obtains all of the market share from the new product nest. This gives the firm an incentive to introduce it if the extra revenue is able to cover the development costs and some loss of demand for its other products. Since the new product also reduces the demand for other firms' varieties, all firms will have an incentive to compete within the new product nest. Hence, the competition for new market share drives firms to produce multiple goods even in the absence of direct cost benefits or the love of variety.

The present work is related to Pavlov and Weder (2017) in the sense that both evaluate the role of multi-product firms in general equilibrium. Here the overlap ends. As mentioned above, they use opposing setups, i.e. market segmentation versus market interlacing, the latter being the central element here that brings multi-product firms to life. As the next section will explain, the elasticities of substitution between products in the CES-bundlers have different economic interpretations across the two frameworks as they pertain to different objects. Pavlov and Weder (2017) concentrate on finding sunspot equilibria and investigate the importance of sunspot shocks for business cycles, whereas here we consider unique dynamics. Accordingly, the present paper simulates the calibrated economy driven by fundamental shocks only.

When considering the implications on aggregate dynamics, market interlacing provides a novel way of generating countercyclical markups, which endogenously amplify the effect of economic shocks. Countercyclical markups in Minniti and Turino (2013) and Pavlov and Weder (2017) arise from their assumptions that firms are large enough to affect the aggregate price index, which is unrelated to the multi-product structure and not required here. There, product scope variations only affect aggregate dynamics in so far as the intra-firm variety effect exists. In contrast, the interlacing setup allows endogenous variations of the product scope and markups without the love of variety and the aggregate price index effects (see Yang and Heijdra (1993), for the latter).

The markup mechanism works through the variation of firms and goods, hence it is advisable to relate it to the literature where markups are negatively related to the number of operating firms. Floetotto and Jaimovich (2008) and Atkeson and Burstein (2008) build models with a continuum of sectors with each sector populated by a finite number of mono-product firms.[4] Firms' price elasticity of demand, and consequently market power, is affected by the entry of new firms when they internalize their price setting effect on the sectoral price level. This parallels our assumption that firms no longer ignore their influence on the product category price index (for example, *Tesla* in the market for electric cars). An important difference is that a firm in our model produces in multiple product categories rather than in a single sector. Furthermore, product scope variations influence entry dynamics, and as Section 2.4 will show, differences in the inter-firm and inter-product elasticities are not only important for markup cyclicality but also for the existence of multi-product firms. Colciago and Etro (2010) and Bilbiie et al. (2012) also discuss an alternative way to generate markups that are countercyclical in the number of firms. They depart from the Dixit-Stiglitz bundling of goods and consider translog preferences. With these preferences, more product variety leads to increased substitutability and thus firm entry lowers the economy's markups (it also lowers the love of variety, which the current paper omits completely).

We then frame this setup in dynamic general equilibrium to study the model's ability to explain the U.S. business cycle and product scope dynamics. We extract shocks from the empirical versions of the labor and efficiency wedges and demonstrate that the model matches well the volatilities and correlations of the key macroeconomic variables. Importantly, our model predicts procyclical product scopes of multi-product firms which aligns well with Guo's (2021) findings, the two wedges are able to explain the change in macroeconomic dynamics between the pre-Great Moderation years and the period since 1984 and we can also account for several labor market facts including the increases of the relative volatilities of hours and labor productivity (see for example Galí and van Rens (2021)) as well as Christiano and Eichenbaum's (1992) hours-productivity puzzle. This puzzle, dating back to Dunlop (1938) and Tarshis (1939), entails that the correlation of hours worked and productivity is slightly negative in the data which is a conundrum for models in the real business cycle tradition. In fact, we can moreover explain that this relationship has turned significantly negative after 1983. Our model further replicates the drop and loss of correlation of total factor productivity with the main macroeconomic variables. While the above stylized facts are not new and have been partially explained by others, we believe to be the first to address them simultaneously, as well as with a quite parsimonious approach.

2. Model

The model economy consists of intermediate sector firms who are able to choose how many distinct goods to produce. These goods are differentiated and hence bring about market power. The commodities are bought by competitive firms that weld them

[3] See Pavlov and Weder (2017) and the Supplementary Appendix on the importance of the love of variety for the market segmentation setup.

[4] Colciago and Etro's (2010) model is similar but they assume a representative sector framework and also compare Bertrand and Cournot competition.

together into the final good that can be consumed or, by adding it to the capital stock, invested. People own the two factors of production and rent out their services on perfectly competitive markets. This section sets out the technology side of product aggregation and in particular the market interlacing structure. We will discuss details on people and introduce fundamental shocks in Section 3 where we will need them to simulate the model.

2.1. Technology

Production occurs in two phases. Final goods are produced by perfectly competitive firms as a combination of N_t product nests with each nest indexed $j \in [0, N_t]$ being a package of differentiated goods produced by $M_t(j)$ intermediate firms. There is a finite number of intermediate goods producing firms, who are not small relative to the size of the market. The inputs produced by these firms are imperfect substitutes and this nature conveys them market power. Let us call each such good $y_t(j,i)$ where j denotes the product supplied and i the firm. The composite of each of these goods (i.e. a nest), coming from the different producers, is

$$Y_t(j) = M_t(j)^{\frac{1}{1-\theta}} \left(\sum_{i=1}^{M_t(j)} y_t(j,i)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \qquad \theta > 1.$$

Here, the composite good $Y_t(j)$ can be interpreted as the value of varieties in a product category, say electric cars.[5] The inter-firm elasticity of substitution θ then determines the degree of differentiation between different firms' products of the same category, say between *Tesla Model 3* and *BMW i4* electric cars. Each of these composites is then welded together to make up final output as in

$$Y_t = N_t^{\frac{1}{1-\gamma}} \left(\int_0^{N_t} Y_t(j)^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \qquad \gamma > 1.$$

Here, the inter-product elasticity of substitution γ determines the differentiation between product types, for example between electric cars and motorcycles.[6] The equilibrium product scope N_t can thus be interpreted as the number of product categories produced in the economy. With this interpretation, we will find it reasonable to assume that goods within the same nest are more substitutable than goods across nests (such as *Tesla Model 3* electric cars versus *BMW R Series* motorcycles), thus, $\theta > \gamma$. We further impose that CES-aggregators eliminate any love-of-variety effects, thus, both of these aggregators are constant-returns-to-scale production functions.[7] Demand for good $y_t(j,i)$ is

$$y_t(j,i) = \left(\frac{p_t(j,i)}{P_t(j)} \right)^{-\theta} \left(\frac{P_t(j)}{P_t} \right)^{-\gamma} \frac{Y_t}{M_t(j) N_t} \tag{1}$$

given the price index for product category j

$$P_t(j) \equiv M_t(j)^{\frac{1}{\theta-1}} \left(\sum_{i=1}^{M_t(j)} p_t(j,i)^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{2}$$

and the aggregate price index

$$P_t \equiv N_t^{\frac{1}{\gamma-1}} \left(\int_0^{N_t} P_t(j)^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}. \tag{3}$$

Firms hire labor, $h_t(j,i)$, and capital services, $k_t(j,i)$, and have access to the production technology

$$\int_0^{N_t(i)} y_t(j,i) dj = \int_0^{N_t(i)} \left[k_t(j,i)^{\alpha} h_t(j,i)^{1-\alpha} - \phi \right] dj - \phi_f \qquad 0 < \alpha < 1, \phi > 0, \phi_f > 0 \tag{4}$$

where $N_t(i)$ is the product range of firm i. The parameter ϕ_f stands for an overhead cost component that applies in each period of an active firm and it is independent of how much output is produced. The term ϕ is specific to each variety that is offered and can be thought of as marketing or development costs. It restricts the amount of varieties offered by firms.

2.2. Market structure and product scope choice

The decisions of an intermediate good firm follow a two-stage game. In the first stage, firms choose how many varieties to produce. In the second, they compete as Bertrand competitors and choose their markups, labor and capital. The model is solved by backward induction using the subgame Nash perfect equilibrium concept. Free entry via a zero-profit condition determines the

[5] In contrast, the composite good in the market segmentation setup of Minniti and Turino (2013) and Pavlov and Weder (2017) consists of varieties produced by a single firm e.g. a *BMW* nest. While θ is the across firm elasticity in both setups, under interlacing it is across individual varieties and under segmentation it is across nests.

[6] In the segmentation setup, γ is the within firm elasticity as it determines the substitution between individual varieties of the same firm, not between nests as is done here.

[7] The variety effect would add an additional amplification mechanism but, unlike in Pavlov and Weder (2017), it is not required for multi-product firms to exist. We considered introducing it in the simulated model in Section 3 but found that it had minimal effect on our results.

number of firms. Since all firms face the same costs, technology and behavior is governed by identical first-order conditions, a symmetric Nash equilibrium emerges every period.

Unlike Dixit and Stiglitz (1977), firms are not small relative to the economy. We design oligopolistic pricing by assuming that firms do not neglect their influence on the product category price index. However, like Atkeson and Burstein (2008) and Floetotto and Jaimovich (2008), we assume that the effect on the aggregate price index is sufficiently small so that it can be ignored.[8] Taking logs of (1) yields

$$\ln y_t(j,i) = -\theta \ln p_t(j,i) + (\theta-\gamma)\ln P_t(j) + \gamma \ln P_t + \ln Y_t - \ln N_t - \ln M_t(j)$$

and using (2) the price elasticity of demand is thus

$$\frac{\partial \ln y_t(j,i)}{\partial \ln p_t(j,i)} = -\theta + (\theta-\gamma)\left(\frac{p_t(j,i)}{P_t(j)}\right)^{1-\theta} M_t(j)^{-1}.$$

Clearly, if firms were to take $P_t(j)$ as given or $\gamma = \theta$, then $\partial \ln y_t(j,i)/\partial \ln p_t(j,i) = -\theta$, and the markup would be a constant as in Dixit and Stiglitz's (1977) approximation. The difference between elasticities θ and γ acts like a weight that amplifies the cyclicality of the markup. Firms maximize profits

$$\pi_t(i) = \int_0^{N_t(i)} p_t(j,i)y_t(j,i) - w_t h_t(j,i) - r_t k_t(j,i)dj$$

subject to production technology (4) while taking factor prices w_t and r_t as given. Optimal factor demands entail

$$w_t = (1-\alpha)\Lambda_t k_t(j,i)^{\alpha} h_t(j,i)^{-\alpha}$$

$$r_t = \alpha\Lambda_t k_t(j,i)^{\alpha-1} h_t(j,i)^{1-\alpha}$$

where Λ_t denotes marginal costs. As in Minniti and Turino (2013) and Pavlov and Weder (2017), the firm ignores the cross-price elasticities of demand and the first-order condition with respect to $p_t(j,i)$ is

$$y_t(j,i) = \left[p_t(j,i) - \Lambda_t\right]\left(\frac{y_t(j,i)}{p_t(j,i)}\theta + \frac{y_t(j,i)}{p_t(j,i)}(\gamma-\theta)\left(\frac{p_t(j,i)}{P_t(j)}\right)^{1-\theta} M_t(j)^{-1}\right).$$

Since all firms face the same factor prices, each will charge the same price for its products. The last equation then rearranges for the markup

$$\mu_t(j,i) \equiv \frac{p_t(j,i)}{\Lambda_t} = \frac{\theta + (\gamma-\theta)\left(\frac{p_t(j,i)}{P_t(j)}\right)^{1-\theta} M_t(j)^{-1}}{\theta + (\gamma-\theta)\left(\frac{p_t(j,i)}{P_t(j)}\right)^{1-\theta} M_t(j)^{-1} - 1} \tag{5}$$

and for any firm i the price of product j is the same, that is $p_t(j,i) = p_t(j,k)$. Clearly if $\theta = \gamma$ then the markup is constant at $\theta/(\theta-1)$. If firm i were the only firm selling product j then $p_t(j,i) = P_t(j)$ and, since $\theta > \gamma$, the firm is able to charge the highest markup at $\gamma/(\gamma-1)$. As we will see next, this provides an incentive for firms to introduce new products but, in general equilibrium, firm entry into market j will drive down the markup. Using the above first-order conditions, profits can be written as

$$\pi_t(i) = \int_0^{N_t(i)} y_t(j,i)\left[p_t(j,i) - \Lambda_t\right] dj - \Lambda_t[N_t(i)\phi + \phi_f].$$

To determine its product line, a firm chooses the length of the N_t -integral while taking the number of active firms and their product scope choices as given.[9] Using (1) and noting from (2) that if only one firm is producing in the new product category then $p_t(N,i) = P_t(N)$, the first-order condition for the product scope, $\partial\pi_t(i)/\partial N_t(i) = 0$, can be written as

$$\Lambda_t\phi = \left(\frac{P_t(N)}{P_t(j)}\right)^{1-\gamma}\left(\frac{P_t(j)}{P_t}\right)^{1-\gamma}\frac{P_tY_t}{N_t}\frac{p_t(N,i)-\Lambda_t}{p_t(N,i)} - \int_0^{N_t(i)}\left(\frac{p_t(j,i)}{P_t(j)}\right)^{1-\theta}\left(\frac{P_t(j)}{P_t}\right)^{1-\gamma}\frac{P_tY_t}{M_t(j)N_t^2}\frac{p_t(j,i)-\Lambda_t}{p_t(j,i)}dj. \tag{6}$$

The left-hand side represents the direct cost of producing a new variety. The first term on the right hand side represents the extra revenue from selling the new variety and the second term the loss of revenue due to the cannibalization effect (new products reducing the demand for existing varieties). This effect not only cannibalizes the demand for firm i's products but also the demand for other firms' products. As noted above, if only one firm is producing in the new product category, then it is able to charge the highest markup at $\gamma/(\gamma-1)$, which implies $p_t(N,i) > p_t(j,i)$. A higher markup in the first term thus gives a greater incentive to introduce a new product (net of the cannibalization effect). In this sense, the reward of higher markups for the N'th good gives firms an incentive to expand their product scopes.

[8] In the Supplementary Appendix, we show that our results remain robust if firms were to internalize the aggregate price index effect.

[9] The product scope has no effect on the prices of varieties or the product category price index (see Appendix A.2).

2.3. Symmetric equilibrium

In a symmetric equilibrium, each firm produces the same number of varieties $N_t(i) = N_t$ and charges the same price p_t. Let us designate the final good to be the numeraire, $P_t = 1$, and therefore from (2) and (3), $p_t = P_t$. The markup simplifies to

$$\mu_t = \frac{\theta M_t + \gamma - \theta}{(\theta - 1)M_t + \gamma - \theta} \tag{7}$$

and the zero-profit condition governs firm entry[10]

$$M_t = \frac{Y_t\left(\mu_t - 1\right)}{N_t\phi + \phi_f}. \tag{8}$$

The markup is negatively related to the number of operating firms. This is reminiscent to Floetotto and Jaimovich (2008) and Colciago and Etro (2010). Noting from (5) that if only one firm is producing in the new product category $p_t(N,i)/\Lambda_t = \gamma/(\gamma - 1)$, and that $P_t(N)/P(j) = 1$ (see Appendix A.3), (6) thus rearranges for the product scope

$$N_t = \frac{Y_t}{\phi}\left(\frac{\mu_t}{\gamma} - \frac{\mu_t - 1}{M_t}\right). \tag{9}$$

As will be shown in the next sections, the product scope is procyclical. Substituting (7) and $Y_t = M_t N_t y_t$ in (9) rearranges for output per variety

$$y_t = \frac{\phi\gamma\left(M_t(\theta - 1) + \gamma - \theta\right)}{\theta M_t(M_t - 1)}$$

which, like in Minniti and Turino (2013) and others, is countercyclical with respect to the number of firms due to the cannibalization effect. Finally, denoting aggregate capital and hours as $K_t = M_t N_t k_t$ and $H_t = M_t N_t h_t$, it is straightforward to obtain aggregate output as

$$Y_t = \mu_t^{-1} K_t^{\alpha} H_t^{1-\alpha}.$$

The endogenous total factor productivity in the model is thus driven by the countercyclical markup. As Section 3 will show, its elasticity is affected by the relationship between entry and product scope.

2.4. Product scope

This section discusses the conditions under which multi-product firms exist even when variety effects are assumed away. This is of importance since Pavlov and Weder (2017) have shown that the intra-firm variety effect in the *market segmentation* case was necessary in order for firms to produce multiple products. To do this, we press into service (7), (8) and (9) so that the product scope can be written as

$$N_t = \frac{\phi_f}{\phi}\frac{\mu_t\theta(\gamma - 1) - \gamma\theta}{\mu_t\theta + \gamma\mu_t - 2\gamma\theta(\mu_t - 1)}.$$

Multi-product firms exist whenever N_t is strictly positive, which is inversely related to variety-specific fixed costs ϕ, i.e. entry costs for a new product. Let us begin with the case of firms being single-good sellers, i.e. by normalization $N_t \to 0$. This case is equivalent to

$$\mu_t = \frac{\gamma}{\gamma - 1} \equiv \mu^{\max}. \tag{10}$$

Here, the number of firms is at its lower limit and this upper bound of the markup corresponds to the situation of a single good and nest produced in the economy, thus, attaining maximum market power for that single firm. The product scope maxes when μ_t is such that

$$\mu^{\max} > \mu_t = \frac{2\theta\gamma}{2\theta\gamma - \theta - \gamma} \equiv \mu^{\min} > \frac{\theta}{\theta - 1} \tag{11}$$

in which case $N_t \to \infty$ and the mass of firms reaches its maximum at $M_t = (\theta + \gamma)/\theta$. If the number of firms were higher then profits would be negative. In the absence of any cost advantages or the love of variety, the markup would be too low to cover the fixed costs required to maintain the product range (it can be shown that as long as the markup is between $\mu^{\min}$ and $\mu^{\max}$ then profits are sufficient to cover the two fixed costs). As all nests are populated by multiple goods and firms, the markup $\mu^{\min}$ is determined by a combination of the inter-firm and the inter-product elasticities of substitution. It is easily conceived from the expression of $\mu^{\min}$ that the two elasticities operate symmetrically. Also, this markup is strictly smaller than $\mu^{\max}$. The minimum markup is greater than $\theta/(\theta - 1)$ which would prevail if firms were to take the price index $P_t(j)$ as given as in the Dixit-Stiglitz approximation of

[10] Results for the existence of multi-product firms do not change by modeling dynamic entry with sunk costs as in Bilbiie et al. (2012, 2019).

monopolistic competition. The implication of (11) is that θ must be sufficiently larger than γ for multi-product firms to exist. Also, it is easily demonstrated that the product scope and the markup are inversely related

$$\frac{\partial N_t}{\partial \mu_t} = \frac{\phi_f}{\phi} \frac{\gamma(\gamma-\theta)}{(\mu_t\theta + \gamma\mu_t - 2\gamma\mu_t\theta + 2\gamma\theta)^2} < 0.$$

Entry of competitors lowers market power and leads to a higher product scope. Since entry reduces the market share and profits from existing varieties, firms will then have an incentive to produce new goods (i.e. establish new product nests) to command potentially greater market power over them. Since all firms face the same constraints and technologies, they all enter the markets for new product categories and the production of existing varieties then drops (i.e. a cannibalization effect). The outcome of entry is therefore lower output per product nest, $M_t y_t$, but a higher output per firm, $N_t y_t$. We have now set up a theory of multi-product firms. We will next explore an economy that is populated by such firms and derive business cycle implications.

3. The statistical behavior of the model

This section asks whether the artificial economy's fluctuations and comovements are consistent with the observed behavior of the analogous data series. Let us begin by embedding people and shocks into the model.

3.1. General equilibrium and wedges

People are represented by an agent with lifetime utility

$$\sum_{t=0}^{\infty} \beta^t u(C_t, H_t)$$

who owns both factors of production and sells their services to the firms. We also introduce variable capital utilization and the agent's intertemporal constraint implies

$$K_{t+1} = (1-\delta_t)K_t + w_t H_t + r_t K_t - C_t$$

in which the physical rate of capital depreciation $\delta_t = \frac{1}{\kappa}U_t^{\kappa}$ is a convex function of the utilization rate U_t, a rate that is decided by the owner of capital. We simulate the artificial economy by unsettling it with sequences of two wedges. While we remain agnostic about the real underlying factors, one may think of them as aggregate supply and demand disturbances but mappings may exist between our model and detailed models with frictions that do not follow this convention.[11] In the model they enter as perturbations to technology and to preferences and in reduced form, the two perturbations parallel the efficiency wedge and the labor wedge. In line with the Brinca et al. (2016) findings, we restrict our analysis to these two wedges since they constitute the main driving forces of fluctuations of the U.S. aggregate economy. Formally, each firm i's production function for product j becomes

$$Z_t \left(U_t k_t(j,i)\right)^{\alpha} \left(g^t h_t(j,i)\right)^{1-\alpha} - g^t\phi$$

where Z_t is a random productivity parameter and g is the gross growth rate of labor augmenting technological progress that affect all firms equally. Preference shocks disturb the marginal rate of substitution between consumption and leisure. We follow Arseneau and Chugh (2012) and Foroni et al. (2018) in that the shocks act on the disutility of work.[12] The labor wedge τ_t derives from the static first-order condition given the agent's period utility function

$$u(C_t, H_t) = \ln C_t - \Delta_t \frac{\upsilon}{1+\chi} H_t^{1+\chi} \qquad \upsilon > 0, \chi \geq 0$$

in which Δ_t is a preference shifter, υ measures the disutility of working and χ is the Frisch labor supply elasticity. The first-order conditions from the agent's utility maximization problem and the equilibrium real wage combine to

$$\Delta_t = \frac{Y_t}{C_t} \frac{1-\alpha}{\upsilon H_t^{1+\chi}} = \frac{1}{1-\tau_t}$$

where the second equality points to the relation with Shimer's (2009) labor wedge.[13] As for the efficiency wedge, from the aggregate production function, the conventional Solow residual adjusted for utilization SR_t implies

$$SR_t = \widehat{Y}_t - \alpha\widehat{K}_t - \alpha\widehat{U}_t - (1-\alpha)\widehat{H}_t = \widehat{Z}_t - \widehat{\mu}_t.$$

where hatted variables denote percent deviations from the steady state. The countercyclical markup gives an upward bias to SR_t as an estimator of the efficiency wedge. Using the log-linearized equations, we can eliminate the markup to yield

$$\widehat{Z}_t = (1+\epsilon_{\mu})\widehat{Y}_t - \alpha\widehat{K}_t - \alpha\widehat{U}_t - (1-\alpha)\widehat{H}_t$$

11 Brinca et al. (2016) discuss various such mappings.

12 See Baxter and King (1991), Bencivenga (1992), Galí and Rabanal (2004) or Weder (2006) for alternative specifications of preference shocks.

13 The 1960–2006 correlation between our annualized wedge and Shimer's is 0.95. Differences are due to slightly differing calibrations and data.

Table 1
Shock standard deviations.

	1948–1983	1984–2019
Efficiency wedge shocks, ε_t^z	0.686	0.459
Labor wedge shocks, ε_t^Δ	2.637	1.945

Standard deviations are in percent terms.

where

$$\varepsilon_\mu \equiv -\frac{\mu\theta + \gamma(\mu + 2\theta - 2\mu\theta)}{\mu(\gamma + \theta - 2\gamma\theta)} < 0$$

is the markup elasticity with respect to output. We calibrate the steady state markup to $\mu = 1.3$, which lies in the proximity of the level estimated by De Loecker et al. (2020), Edmond et al. (2021) and others. The elasticity of substitution between products within a nest is set at $\theta = 10$ and this value is close to the midpoint between the two elasticities estimated by Broda and Weinstein (2010). Then, Eqs. (10) and (11) imply that γ must satisfy

$$\frac{\theta\mu}{2\theta(\mu-1)-\mu} < \gamma < \frac{\mu}{\mu-1}$$

and we set $\gamma = 3.5$, which is approximately halfway between the inequality's bounds. We assign $\chi = 2$ for the Frisch labor supply elasticity. This value aligns with Chetty et al.'s (2013) recommendations for calibrating macroeconomic models. We further set $g = 1.0045$ which helps to pin down $\kappa = (g/\beta - 1 + \delta)/\delta$. Finally, to make comparisons with previous studies straightforward: $\alpha = 0.3$, $\delta = 0.025$ and $\beta = 0.99$. The above is then used to back out the empirical sequences of Z_t and Δ_t from observable data.[14]

Using U.S. data ranging from 1948:II to 2019:IV, we find that the wedges appear to undergo several changes in regards to their cyclical pattern. When reporting is based on Hodrick–Prescott filtered time series with a smoothing parameter $\lambda = 1600$, the efficiency wedge's standard deviation drops slightly when comparing pre- and post-1983 periods (1983:IV is the cut-off quarter, coinciding with the onset of the Great Moderation).[15] Also, the efficiency wedge's contemporaneous correlation with output goes from 0.22 to −0.29 for the second period. This correlation shift suggests a smaller importance of technology shocks. In fact, it parallels the trend in U.S. labor productivity to hours correlation which became strongly countercyclical, from −0.30 to −0.82 as you can see from Tables 3 and 4. The labor wedge τ_t does not go through such shifting pattern as its output correlation remains constant at −0.83.[16] Both wedges are highly persistent which translates to our calibration of the parameters $\rho_z = 0.970$ and $\rho_\Delta = 0.985$ and by restricting the driving processes to AR(1), for example, the labor supply shock follows[17]

$$\ln \Delta_t = 0.985 \ln \Delta_{t-1} + (1 - 0.985) \ln \Delta + \varepsilon_t^\Delta.$$

As you can see from Table 1, both shocks series have become more subdued as their standard deviations drop in the post-1983 period.

3.2. Aggregate fluctuations

To assess aggregate fluctuations as seen through the lens of our model, we feed back in the sequence of shocks $\left\{\varepsilon_t^z, \varepsilon_t^\Delta\right\}_{1948:II}^{2019:IV}$ into the log-linearized artificial economy. Table 2 reports data and artificial second moments for the pre-Great Moderation as well as for post-1983 periods, where TFP stands for naively-measured total factor productivity i.e. the basic Solow residual unadjusted for utilization and market power. Everything has been Hodrick–Prescott filtered and in terms of the key macroeconomic aggregates the model replicates well the relative standard deviations as well as the moderation of volatilities across the two subperiods. The model also predicts a fall in the volatility of utilization. The volatility of labor productivity has remained basically unchanged. The artificial economy correctly mimics this pattern. The model also reproduces the increase of the standard deviation of hours relative to output by around 50 percent as well as the doubling of the standard deviation of labor productivity relative to that of output.[18] Through the lens of our theory, and as in Arias et al. (2007), the post-1983 decline in business cycle volatility comes about from a lower volatility of shocks. However, the composition of shocks – the labor wedge shocks become relatively more important in particular since the mid 2000s – plays a key role for various facts as you will see next.

Tables 3 and 4 lay out cross-correlations for data (upper triangle) and model (lower triangle), where X denotes investment. In addition to matching correlations across the board, three features that relate to the dynamics of the labor market jump out. First, our artificial economy provides a parsimonious solution to the Dunlop–Tarshis puzzle, dating back to Dunlop (1938) and Tarshis (1939), that the correlation of hours worked and productivity is slightly negative in the data which is a conundrum for models in

[14] The correlation between our efficiency wedge and Fernald's (2014) utilization adjusted counterpart over the entire sample is 0.81 — the slight discrepancy is due to the differences in technologies, the presence of market power and the calibration.
[15] We have also used the Hamilton (2018) filter and our main results remain unchanged.
[16] Appendix A.4 provides further details.
[17] We also considered allowing for cross-correlated wedges but decided to keep our presentation parsimonious.
[18] See also Champagne and Kurmann (2013) and Stiroh (2009).

Table 2
Standard deviations.

	1948–1983		1984–2019	
	U.S.	Model	U.S.	Model
Output	2.02	1.94	1.01	1.06
Consumption	0.95	0.95	0.70	0.55
Investment	5.21	5.53	4.28	2.98
Hours	2.10	2.01	1.62	1.61
Labor productivity	0.91	0.99	0.93	1.01
Utilization	1.76	1.22	0.98	0.70
TFP	0.99	1.01	0.65	0.73

Table 3
1948–1983 cross correlations.

	Y	C	X	H	Y/H	U	TFP
Y	1	0.76	0.78	0.90	0.14	0.84	0.71
C	0.97	1	0.60	0.75	−0.02	0.59	0.46
X	0.99	0.93	1	0.77	−0.05	0.71	0.46
H	0.87	0.93	0.82	1	−0.30	0.75	0.34
Y/H	0.18	0.01	0.34	−0.32	1	0.14	0.79
U	0.97	0.89	0.99	0.81	0.24	1	0.63
TFP	0.68	0.52	0.76	0.25	0.83	0.74	1

Table 4
1984–2019 cross correlations.

	Y	C	X	H	Y/H	U	TFP
Y	1	0.80	0.87	0.85	−0.40	0.71	0.09
C	0.95	1	0.65	0.69	−0.34	0.51	−0.00
X	0.98	0.87	1	0.90	−0.63	0.68	−0.20
H	0.79	0.91	0.68	1	−0.82	0.63	−0.44
Y/H	−0.22	−0.45	−0.06	−0.77	1	−0.33	0.86
U	0.97	0.86	0.99	0.73	−0.16	1	0.09
TFP	0.22	−0.04	0.39	−0.41	0.89	0.30	1

the real business cycle tradition. In fact, this relationship has turned significantly negative after 1983. The idea that a combination of business cycle shocks can resolve the puzzle was first entertained by Christiano and Eichenbaum (1992). However, for obvious reasons, they do not address the Great Moderation and also fail to drive down the hours-productivity correlation in the model to a negative. Our model not only solves the puzzle but it also replicates the increasingly more negative correlation that you can observe for the post-1983 data (from −0.30 to −0.82). Second, the correlation of labor productivity and output in the model drops from 0.18 to −0.22 which parallels the fall of 0.14 to −0.40 in the data. This fall lines up with the findings of, for example, Galí and Gambetti (2009), Hagedorn and Manovskii (2011) and Galí and van Rens (2021) regarding a changed cyclicality of labor productivity. Similar to Barnichon (2010) and Garin et al. (2018), our work speculates that this change reflects shifts to the composition of shocks. The intuition can be framed in the model's spot labor market. The efficiency wedge knocks around the labor demand curve and the labor wedge shifts labor supply. If fluctuations of the labor wedge become more prevalent then labor productivity becomes less procyclical as the driver of this procyclicality grows less important — in a sense, movements along the labor demand schedule are the dominant shifts. The same change to the shock composition turns the correlation of productivity and hours to more negative precincts. Thirdly, while this parallels Arias et al. (2007), unlike in their model, here the correlation of TFP with the main macroeconomic variables changes considerably as it does in U.S. data.[19] For example, the correlation of TFP and employment falls from 0.46 to −0.44 in the US economy and it falls from 0.25 to −0.41 in our artificial economy. In sum, while different aspects of cyclicality and changes thereof can be explained by other theories, we see our approach as having three main advantages: (i) we do not require changes to the structural model parameters to account for facts and (ii) we are able to explain central elements of labor market dynamics but moreover also a flock of key macroeconomic patterns and (iii) we are able to do this in a very parsimonious way with only two shocks.

Table 5 confirms that the model fits the data well by presenting the correlations between the artificial and U.S. series. In particular, output, hours and labor productivity are almost perfectly correlated. This finding, in combination with the model's ability to fluctuate nearly the same amplitude as the U.S. economy, suggests that indeed the two wedges, i.e. labor and efficiency, explain the bulk of U.S. aggregate fluctuations.

Another way to judge the success of the model in replicating business cycles is mapping output's cross-correlation over time. We do this by calling into play a classical method of business cycle analysis developed by Burns and Mitchell (1946). It allows to visualize

[19] Arias et al. (2007) compute shocks differently due to different assumptions regarding preferences and technology as well as perfect market structure. They also do not use actual shock sequences taken directly from the two wedges.

Table 5
Data vs model correlations.

	Y	C	X	H	Y/H	U	TFP
1948–1983	0.96	0.85	0.71	0.99	0.92	0.74	0.93
1984–2019	0.95	0.87	0.72	0.99	0.98	0.58	0.94

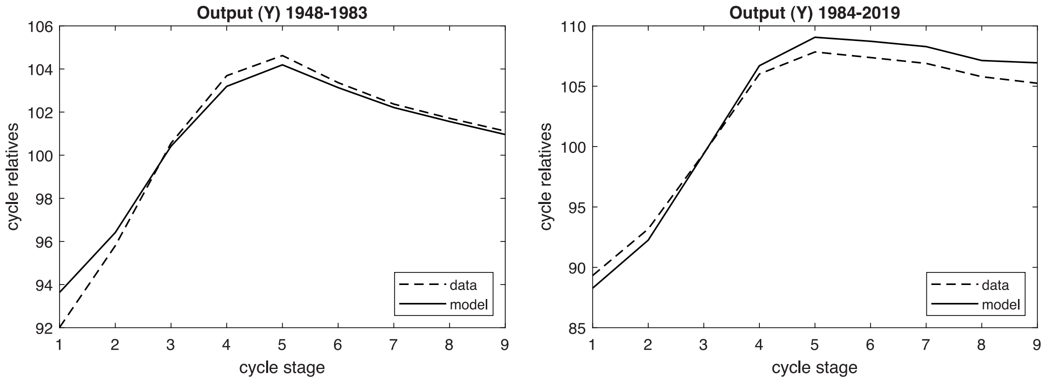

Fig. 1. Average behavior of aggregate output.

the average business cycle for the pre- and post-83 subsamples, as well as the lead–lag structure of aggregate fluctuations.[20] Fig. 1 presents the average cycles for real per capita artificial and U.S. GDPs. The cycle relatives denote the percentage of the complete cycle mean across nine stages with stage 5 being the peak based on NBER dating. The similar general shapes of the data and model series demonstrate that the artificial series of aggregate output matches well the post-war U.S. cycles. The artificial and data GDPs exhibit a distinct procyclical pattern, rising during expansions and more slowly falling during contractions and both doing so at very similar amplitudes. Both series peak at the same stage. Moreover, the slight changes in the pre/post-peak patterns that occur between periods is well replicated by the model as well.

3.3. Product scope, firm dynamics and markup fluctuations

A central part of our model is the role of the time-varying product scope, firm net entry and markups. Regarding the procyclicality of the entry rate, evidence abounds, for example Portier (1995), Lewis (2009) and Lee and Mukoyama (2015, 2018). There is a long list of research suggesting a countercyclical markup and Bils et al. (2018) is one representative.[21] Guo (2021) presents evidence for a procyclical product scope pattern by applying Nielsen Retail Scanner data of U.S. consumer goods purchases.[22] In the spirit of her paper, we run regressions of the product scope and two measures of aggregate business cycle activity. In particular, we take time series from our artificial economy when simulated using actual preferences and technology shocks from the 2007:I to 2014:IV period, which matches Guo's, and find that growth rates (product scope, consumption and utilization) have the following relationships:

$$\Delta \ln N_t = \underset{(4.58)}{0.004} + \underset{(19.01)}{3.532}\Delta \ln C_t \qquad \overline{R}^2 = 0.91$$

$$\Delta \ln N_t = -\underset{(-13.83)}{0.005} + \underset{(35.55)}{2.298}\Delta \ln U_t \qquad \overline{R}^2 = 0.97$$

where t-values are in parentheses. Clearly, the coefficients are not directly comparable to Guo's (2021) since she places firms into different categories, however, our model produces a procyclical pattern similar to her empirical findings. Table 6 presents the Hodrick–Prescott filtered standard deviations for the product scope, the number of firms, and the markup for the 1948–1983 and 1984–2019 periods. Once again, all series have become less volatile after 1983. Net firm entry and product scope are strongly procyclical and the markup moves countercyclically. The product scope is approximately one and a half times more volatile than output and the volatility of firms is only one tenth. The fact that product scope is more volatile than net firm entry is compatible with Guo (2021) and Broda and Weinstein (2010). In our model, since firms compete in all product categories, profits are eroded quickly and potential competitors are deterred from entry.[23] The low volatility of the number of firms implies a relatively smooth countercyclical markup that acts as a mild amplification mechanism.

[20] A brief description of their idea is in Appendix A.5. Adelman and Adelman (1959), Simkins (1994) and King and Plosser (1994) apply the methodology to various business cycle models.

[21] Nekarda and Ramey (2020) counter this apparent consensus.

[22] Looking at products in a different angle, this procyclical pattern is related to Decker et al. (2016) who report that firms increase the set of markets they are serving in booms.

[23] The low volatility of entry here relative to Minniti and Turino (2013) and Pavlov and Weder (2017) can be seen as consistent with Brander and Eaton's (1984) theoretical findings that the interlaced structure is a more competitive structure that deters entry.

Table 6
Firm dynamics and markups.

Variable, x	1948–1983		1984–2019	
	σ_x	$\rho(x, Y)$	σ_x	$\rho(x, Y)$
N	3.12	1.00	1.71	1.00
M	0.23	1.00	0.13	1.00
μ	0.09	−1.00	0.05	−1.00

4. Conclusion

This paper examines multi-product firm dynamics taking a market interlacing industry configuration to general equilibrium. In contrast to previous studies that frame multi-product firms in a market segmentation setup, we show within a market interlacing platform that firms produce multiple products even in the complete absence of the love of variety effects. The model implies a procyclical product scope which agrees with Guo's (2021) empirical findings. In addition, entry and exit of firms provide an endogenous amplification mechanism via a time-varying markup. When simulated by shocks derived from empirical efficiency and labor wedges, the model predicts procyclical product scopes and net firm entry. As a result of a transformation in the empirically observed composition of the wedges' volatilities, the simulated model replicates the changes in aggregate dynamics between the pre-Great Moderation era and the post-1983 period and explains various labor market facts including the hours-productivity puzzle and the increases of the relative volatilities of hours and labor productivity.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

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