Impact analysis of the traffic convoy system and toll pricing policy of the Suez Canal on the operations of a liner containership over a long-haul voyage

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Abstract

This paper takes an initiative to quantitatively assess the impact of the traffic control scheme and the stepwise toll pricing policy of the Suez Canal on the optimal sailing schedule of a liner containership. We first develop a mixed-integer nonlinear programming (MINLP) model for the optimal sailing schedule of a containership over a long-haul voyage via Suez subject to the traffic convoy system of the Suez Canal and its piecewise transit due structure. To improve the computational performance and take advantage of off-the-shelf optimization solvers, we linearize the nonlinearity in clock-time calculation, reformulate the power function in bunker fuel calculation with the second order cone programming technique, and cast the MINLP model into a mixed-integer second order cone programming (MISOCP) model. Various impact analyses can be carried out using the MISOCP model. A case study on a 13000-TEU containership running on the LP4 service operated by APL shows several managerial insights: (a) ignoring the traffic control system at Suez in ship speed optimization may lead to an infeasible sailing schedule, and underestimate the operating cost (even the bunker cost) of a containership on a long-haul voyage via Suez; (b) the optimal ship recovery plan in terms of its sailing speeds is mainly determined by the predefined port time windows, delay situation and Suez-clock time, but not pretty much affected by the levels of bunker price and transit due.

Keywords: Suez Canal; transit due; containership; speed optimization; second order cone programming

1. Introduction

The Suez Canal plays a pivotal role in the global container liner shipping network. Table 1 shows its traffic volumes from 2012 to 2016. Totally 16,833 ships made full transits through the Suez Canal in 2016, out of which 5,414 ships (close to a third) are containerships. The canal conveyed 819.2 million tons of cargo for the north-south and south-north trade in 2016, and containerized cargo, 440.0 million tons, represents about 54% of this total volume. If the equivalence “1 TEU = 11 tons” is adopted (Notteboom, 2012), the containerized cargo volume in 2016 is estimated to 40 million TEUs. The dominant portion of these container flows can be attributed to Europe-Asia trade routes. Notteboom (2012) reports that nearly 93% of the container flows via Suez in 2010 are related to Europe-Asia trade routes.

Table 1. Traffic volumes of the Suez Canal in 2012 – 2016

	2012	2013	2014	2015	2016
# Ships making full transits	17224	16596	17148	17483	16833
# Containerships	6332	6014	6128	5433	5414
Net tonnage (million tons)	928.5	915.5	962.7	998.7	974.2
Cargo volume (million tons)	739.9	754.5	822.3	822.9	819.2
Containerized cargo volume (million tons)	398.0	406.0	435.0	428.7	440.0
Containerized cargo share	54%	54%	53%	52%	54%

Note: source – Suez Canal Authority Yearly and Monthly Reports (SCA, 2017).

The traffic control rules at the Suez Canal pose strict restrictions on the transit behaviours of ships through the canal. There are totally two convoys of ships scheduled to transit the canal in one day: one for southbound (labelled as “N Convoy” by the Suez Canal Authority) and the other for northbound (labelled as “S Convoy”). The transit time of a ship through the canal is about 11-14 hours. N Convoy begins to transit the canal at the time 03:30 every day, while S Convoy begins the transit in the opposite direction at the time 04:00. If the ships in the northbound convoy meet those in the southbound convoy during transit, some predesigned waiting or ship collision-avoidance rules and facilities at the Great Bitter Lake and Ballah bypass will be activated (SCA, 2017). This two-convoy system might cause the long waiting times of ships before transit. Let us consider a 13000-TEU liner containership operated over the LP4 service by APL (American President Lines) from Singapore to Le Havre, as shown in Fig. 1. If it is not well scheduled and arrives at Suez at the time 05:00, it has to wait 23 hours for next northbound convoy beginning at 04:00 on the second day. In case the liner ship experienced severe delays due to bad weather/sea conditions from Singapore to Suez, this long waiting time at Suez before transiting the canal will further tighten its shipping schedule from Suez to Le Havre in order to catch up the port time window (or ETA, expected time of arrival) at Le Havre, which definitely requires high sailing speeds and brings a significant increase of bunker fuel consumption.

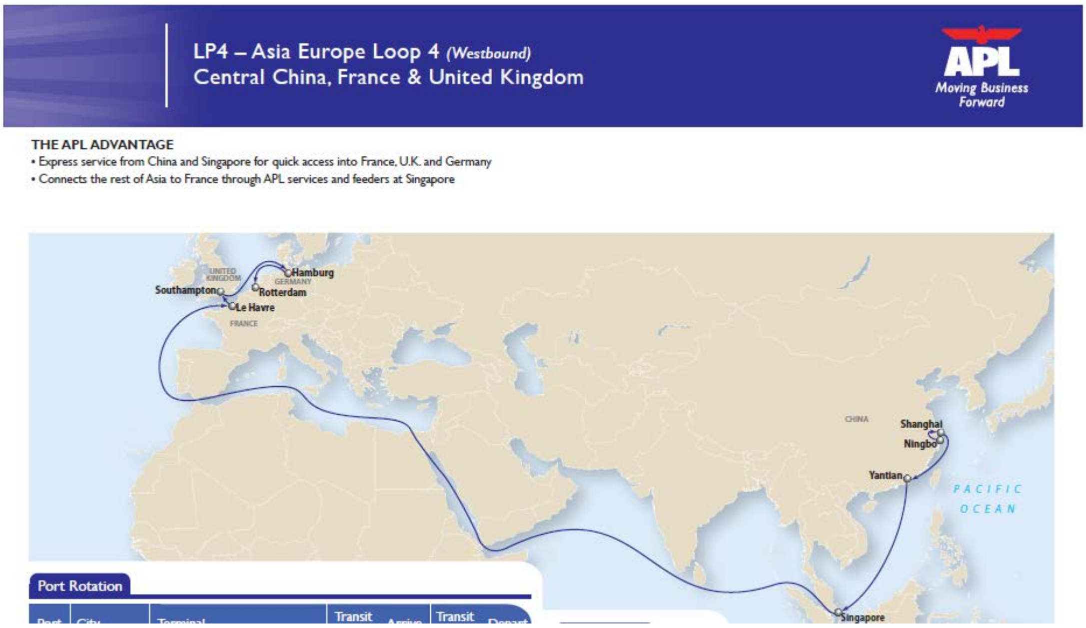

Fig 1. The westbound of LP4 service operated by APL (Source: www.apl.com)

Besides the two-convoy traffic control scheme, the toll pricing policy of Suez may also influence the operating cost of a containership. To set aside enough time for manoeuvring the ships to form a convoy, the Suez Canal Authority (SCA) sets a dead-time limit for each convoy (4-5 hours ahead of the transit beginning time of the convoy). If a ship arrives at Suez before the convoy dead-time limit, it can freely join this convoy by paying the normal transit due. Otherwise, SCA will additionally claim 5-12% of the normal transit due as the surcharge if the ship wants to join this convoy. For a 13000-TEU containership running over service LP4, its normal transit due reaches 422,175 SDR (Special Drawing Rights), roughly 600,000 USD, and its transit surcharges are thus not ignorable if it is not well scheduled.

The purpose of this study is to quantitatively evaluate the impact of the traffic convoy system and toll pricing policy of the Suez Canal on the sailing schedule and operating cost of a liner containership. To fulfil this research purpose, we have to determine the optimal sailing schedule of a liner containership over a long-haul voyage via Suez by minimizing its main operating cost: bunker fuel cost and transit due.

1.1 Literature review

The optimal sailing schedule problem proposed above is close to ship sailing speed optimization, which has been well recognized by recent maritime studies because ship sailing speed is the main determinant of bunker fuel consumption of a ship and bunker cost represents a large portion of ship operating cost. Even the slumps in bunker fuel prices from Nov. 2014 did not change the enthusiasm of container shipping lines for reducing bunker fuel consumption. Generally, higher sailing speed means shorter sailing time and fewer ships required to maintain a fixed, usually weekly, service frequency. Sailing speed optimization is a core of a wide class of issues in container liner shipping network analysis (Christiansen et al., 2012), such as network design (Brouer et al., 2014; Angeloudis et al., 2016), ship fleet deployment (Álvarez, 2009; Ng, 2015; Wang, 2013; Xia et al., 2015), schedule design and recovery (Brouer et al., 2013; Qi & Song, 2012; Li et al., 2016), container assignment (Bell et al., 2011, 2013; Wang et al., 2015), and cargo booking and routing (Song & Dong, 2012). Some work considers schedule design and speed optimization at the *tactical* level and analyzes the relationship between sailing speed/schedule and service frequency and/or fleet deployment (Álvarez, 2012; Du et al., 2017; Notteboom & Vernimmen, 2009; Ronen, 2011). A few studies also address the *operational* sailing schedule (speed control) given the predesigned port time windows (Brouer et al., 2013; Fagerholt et al., 2010; Qi and Song, 2012). For the recent review on ship sailing speed optimization, we refer readers to the work of Psaraftis & Kontovas (2013).

However, studies on shipping network analysis accounting for the influence of the Suez Canal are scant. Brown et al. (1987) optimally determine the schedules of oil tankers for an oil company which also consider ship speed optimization and the transit due at the Suez Canal. Sherali et al. (1999) deal with a similar problem which involves route choice between the routes through the Suez Canal and those around the Cape of Good Hope by a trade-off between travel time and canal due. Brouer et al. (2014) count the canal due in if a sailing link traverses the Suez/Panama Canal in the liner shipping network design problem. However, these studies all address the problems at the strategic or tactical level and thus generally ignore the impact of the traffic convoys at Suez and possible transit due surcharges on shipping schedule.

Some studies on maritime economics also consider the Suez Canal when they investigate the competitiveness of several main shipping routes around the globe. Verny and Grigentin (2009) evaluate the economic feasibility of regular container transport along the Northern Sea Route (NSR) from the viewpoint of cost analysis, in which the operation of a traditional route through the Suez Canal is employed as the benchmark. Schøyen and Bråthen (2011) conduct the similar empirical studies for bulk shipping and adopt 14.4 knots as the speed of ships over the route through the Suez Canal. Notteboom (2012) assesses the market potential of the Cape route vis-à-vis the Suez route by using a distance analysis, a transit time analysis and a generalized cost analysis for a large set of O/D port pairs. This research stream focuses on the cost analysis at the strategic level in order to evaluate the feasibility or profitability of a shipping route, and thus does not go into the operational details on the impact of traffic convoy system at Suez on shipping schedule.

1.2 Objective and contributions

According to the background introduced before Section 1.1 and the above literature review, we can see a gap between industrial needs and academic research. This study aims to make the first move to (but not completely) fill this gap, by modelling the optimal sailing schedule problem for a liner containership and providing some basic managerial insights into the impact of the Suez Canal convoy system and toll pricing policy on ship sailing schedule.

We start from the pricing policy of transit due and the quantitative relationship between sailing speed and bunker fuel consumption, and then develop a mixed-integer nonlinear programming (MINLP) cost minimization model for the optimal sailing schedule problem. To take advantage of off-the-shelf optimization solvers, we linearize the nonlinear “mod” operators, treat the nonlinearity in bunker fuel calculation with the state-of-the-art second order cone programming (SOCP) technique, and finally reformulate the whole model as a mixed-integer SOCP (MISOCP) model with high computational performance.

The contributions of this study are twofold. First, we model the traffic convoy system and toll pricing policy of the Suez Canal in sailing schedule optimization of a liner containership at the operational level. Second, experimental findings in this study provide several intriguing managerial insights on liner containership scheduling over a long-haul voyage: (a) ignoring the ship traffic control of the Suez Canal in sailing schedule optimization might underestimate the operating cost, even the bunker fuel cost, of the ship; (b) the predefined (tactical) schedule, delay situation and clock time at Suez jointly determine the optimal sailing schedule (recovery plan) and main operating cost of a containership over a long-haul voyage via Suez.

The remainder of this paper is organized as follows. Section 2 proposes the research problem. Section 3 builds a MINLP model and reformulates it into a MISOCP one. Section 4 performs the impact analysis of the Suez Canal via a case study and reports our experimental findings, based on the data collected from a global container shipping line. Section 5 draws some concluding remarks. For ease of reading, we tabulate the mathematical notations used in this study in Appendix A. The notations not listed in Appendix A denote the auxiliary variables assisting mathematical transformations (i.e. linearization and SOCP reformulation).

2. Problem Description

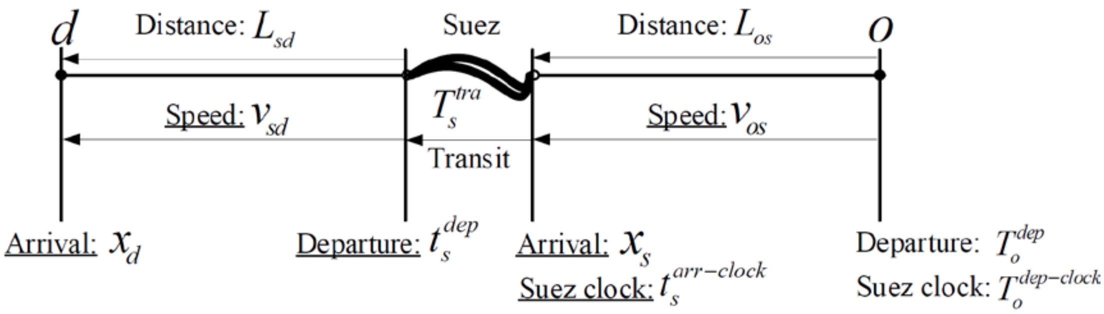

Fig 2. Schematic Representation of ship sailing schedule optimization over a long-haul voyage via Suez

Consider a liner containership deployed on an Asia-Europe shipping service. It is assumed that the containership is now running over a long-haul voyage via the Suez Canal, e.g. from Singapore to Le Havre illustrated in Fig. 1. The optimal sailing schedule problem of the containership over the long-haul voyage can be described as follows with the aid of Fig. 2. The ship leaves its current position o (a port or a waypoint) at time T_o^{dep} (the beginning time of the whole voyage is regarded as time zero) when the clock time at Suez is $T_o^{dep-clock}$, for its long-haul destination d via the Suez Canal. Given the distance from its current position to Suez L_{os} (n miles), the distance from Suez to its destination L_{sd} (n miles) and the transit time through the Suez Canal T_s^{tra}, the shipping line has to determine the optimal sailing schedule over this long-haul voyage for this ship: arrival time at Suez x_s and arrival time at its destination x_d, which are governed by its sailing speeds v_{os} and v_{sd} over the two legs connected by Suez. In this sailing schedule optimization problem, the shipping line should account for the available speed range $\left[\underline{V}, \overline{V}\right]$ of this liner containership and ensure the predefined schedule at destination $\left[\underline{T}_d^{arr}, \overline{T}_d^{arr}\right]$. Meanwhile, according to the ship traffic control schemes of SCA (SCA, 2017), there is only one northbound convoy every day for containerships beginning transit at the time 04:00 (Fig. 3). A containership joining the convoy on a given day usually should arrive at Suez before 23:00. In this case, SCA charges its normal transit due D^{nor}; if it arrives between 23:00 and 00:00, SCA will additionally claim 5% of D^{nor} as the surcharge with a maximum of 12,500 SDR; if it arrives between 00:00 and 01:00, the surcharge is 10% of D^{nor} with a maximum of 25,000 SDR. For arrival time between 01:00 and 04:00, the surcharge is 12% of D^{nor} but with a cap of 30,000 SDR; it will not be admissible to the convoy for arrival later than 04:00. The objective of this sailing schedule optimization problem confronting the shipping line is to minimize the bunker fuel cost of the ship over this long-haul voyage and its transit due at Suez.

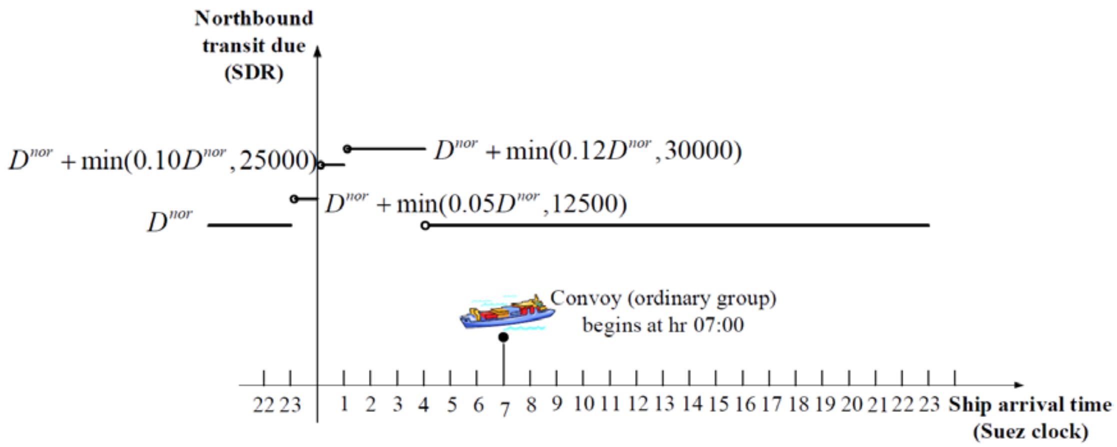

Fig 3. Northbound transit due structure of a containership at Suez

Several concepts are further clarified: (i) A *long-haul voyage via the Suez Canal* in a service is defined as the voyage from the last port of the service in one continent to the first port of the service in another continent, during which the Suez Canal has to be transited; (ii) the problem is to determine the *operational schedule* of the containership, while the predefined schedule designed at the tactical level in form of port time windows, e.g. $\left[\underline{T}_d^{arr}, \overline{T}_d^{arr}\right]$, is referred to as a *tactical schedule*; and (iii) there are two time coordinates involved: the shipping line regards the beginning time of the whole voyage as time zero and increments its count as the ship sail from one position to another over the round voyage, while SCA calculates the transit surcharge of the ship as per the local clock time.

As illustrated by Fig. 3, without loss of generality, we consider a long-haul voyage with northbound transit at Suez. As for a long-haul voyage with southbound transit at Suez, the sailing schedule optimization problem can be similarly formulated.

Our research purpose is to conduct quantitative impact analysis of the traffic convoy system and toll pricing policy at Suez on the optimal sailing schedule of a containership over a long-haul voyage, in order to answer several interesting questions regarding this issue, including

(Q1) Will additional consideration of the Suez Canal traffic system have a significant impact on the optimal sailing schedule and operating cost structure of a containership running on a long-haul voyage via Suez?

(Q2) What is the impact of the Suez Canal on the recovery plan of a liner containership over a long-haul voyage if it experienced a schedule delay, e.g. from Singapore to a waypoint before Suez? How does the delay situation influence its optimal recovery plan and operating cost structure?

(Q3) Will bunker price fluctuation and/or transit due adjustment by SCA substantially influence the optimal sailing schedule and operating cost of a containership over a long-haul voyage via Suez?

To fulfil our research purpose, a mathematical model has to be built for the optimal sailing schedule of a containership with consideration of the traffic convoy system and toll pricing policy at Suez. Next section will mathematically formulate this optimization problem. For modelling purpose, some reasonable assumptions are made in this study:

(A1) The transit time T_s^{tra} of the ship through the Suez Canal is a constant although it varies between 11 and 14 hours in practice. In numerical experiments, it is conservatively set as 14 hours. This assumption will not influence the experimental findings, since it does not change the sailing schedule situation of the ship from origin to Suez and the sailing time difference caused from Suez to destination (at most 3 hours) is negligible.

(A2) The bunker fuel consumption during canal transit of the ship is not counted in. In reality, transit speed of the ship at Suez does not exceed 8.6 knots and the difference in bunker consumption between any two transits of the same ship is trivial.

(A3) For the operating cost of the ship over a long-haul voyage, only bunker cost and transit due at Suez are considered, since they are dominant in cost structure. Other cost components are ignored.

(A4) Extreme congestion at Suez is excluded from the consideration of this study. In other words, the ship can join a traffic convoy as long as time permits and it is willing to, and it will not be delayed to the next convoy due to canal congestion. Convoy delay due to canal congestion seldom occurs at Suez in reality nowadays.

3. Mathematical Programming Model Building

In this section, we first build a MINLP model for the proposed optimal sailing schedule problem. Then we proceed to transform the MINLP model into an equivalent MISOCP model that can be efficiently solved by the commercial optimization solvers such as CPLEX available for the liner shipping industry.

3.1 A mixed integer nonlinear programming model

We first calculate the waiting time of the containership before canal transit, departure time of the containership from Suez and the corresponding transit due. Given the departure time of the ship from origin T_o^{dep}, the corresponding Suez clock time $T_o^{dep-clock}$ and the decision variable x_s (arrival time at Suez), the arrival clock time of the ship at Suez can be expressed by

$$t_s^{arr-clock} = \left[T_o^{dep-clock} + \left(x_s - T_o^{dep} \right) \bmod 24 \right] \bmod 24 \tag{1}$$

where $\left(x_s - T_o^{dep} \right) \bmod 24$ calculates the clock-time increment due to the sailing of the ship from origin to Suez, while the second "mod" deals with the possibility that the value of $\left[T_o^{dep-clock} + \left(x_s - T_o^{dep} \right) \bmod 24 \right]$ is larger than or equal to 24.

To calculate the waiting time of the containership at Suez before transit and the canal due, we introduce three auxiliary binary variables associated with $t_s^{arr-clock}$:

y_1: a binary variable indicating whether $t_s^{arr-clock}$ is between 00:00 and 01:00 hr;

y_2: a binary variable indicating whether $t_s^{arr-clock}$ is between 01:00 and 04:00 hr;

y_3: a binary variable indicating whether $t_s^{arr-clock}$ is between 04:00 and 23:00 hr.

The relationship between these binary indicators and $t_s^{arr-clock}$ can be defined by the following constraints:

$$t_s^{arr-clock} \le 1 + M\left(1 - y_1\right) \tag{2}$$

$$1 - M\left(1 - y_2\right) < t_s^{arr-clock} \le 4 + M\left(1 - y_2\right) \tag{3}$$

$$4 - M\left(1 - y_3\right) < t_s^{arr-clock} \le 23 + M\left(1 - y_3\right) \tag{4}$$

$$y_1 + y_2 + y_3 \le 1 \tag{5}$$

$$y_1, y_2, y_3 \in \left\{0,1\right\} \tag{6}$$

According to the traffic control scheme of northbound transit at Suez, if the containership arrives at Suez before 04:00 hr, it can freely join the current convoy. Otherwise, it has to wait at the anchorage area for the convoy on the next day. The waiting time t_s^{wait} of the ship at Suez before transit (hours) can thus be calculated by

$$t_s^{wait} = \begin{cases} 4 - t_s^{arr-clock}, & 0 \le t_s^{arr-clock} \le 4 \\ 4 + 24 - t_s^{arr-clock}, & 4 < t_s^{arr-clock} < 24 \end{cases} \tag{7}$$

Eq. (7) can be expressed by the following two constraints involving the binary variables defined above:

$$4 - t_s^{arr-clock} - M\left(1 - y_1 - y_2\right) \le t_s^{wait} \le 4 - t_s^{arr-clock} + M\left(1 - y_1 - y_2\right) \tag{8}$$

$$28 - t_s^{arr-clock} - M\left(y_1 + y_2\right) \le t_s^{wait} \le 28 - t_s^{arr-clock} + M\left(y_1 + y_2\right) \tag{9}$$

Now the departure time of the ship from Suez can be calculated as the sum of its arrival time at Suez, waiting time before transit and transit time through the canal, mathematically,

$$t_s^{dep} = x_s + t_s^{wait} + T_s^{tra} \tag{10}$$

As per the toll pricing policy of northbound transit at Suez, when the possible surcharge is taken into account, the transit due f^{tra} (USD) charged by SCA can be expressed as a linear function of the binary variables y_1, y_2 and y_3:

$$f^{tra} = E^{sdr} \begin{bmatrix} D^{nor} + \min\left(0.10D^{nor}, 25000\right) \cdot y_1 + \min\left(0.12D^{nor}, 30000\right) \cdot y_2 \\ + \min\left(0.05D^{nor}, 12500\right) \cdot \left(1 - y_1 - y_2 - y_3\right) \end{bmatrix} \quad (11)$$

where E^{sdr} is the exchange rate (constant) from SDR to USD.

Next, we calculate the bunker cost of the ship over the long-haul voyage. Sailing speed is the main determinant of fuel consumption rate (MT/h, or MT/day) of a ship. The fuel consumption rate (r_F) of a ship is approximately proportional to sailing speed (v) raised to the power β, i.e.,

$$r_F = \alpha \cdot v^{\beta} \quad (12)$$

where α and β are two coefficients to be calibrated using real data. The cubic law, $\beta = 3$, is widely adopted in maritime studies (MAN Diesel & Turbo, 2004; Psaraftis & Kontovas, 2013). To improve calculation accuracy, this study calibrates the coefficients based on the data collected from a global container shipping line. Here, we denote the two coefficients for the leg from origin to Suez by α_{os}, β_{os}, and those for the leg from Suez to destination by α_{sd}, β_{sd}.

The sailing times over the two legs of the long-haul voyage can be calculated as $\left(x_s - T_o^{dep}\right)$ and $\left(x_d - t_s^{dep}\right)$, respectively. The sailing speeds over these two legs are $L_{os} \big/ \left(x_s - T_o^{dep}\right)$ and $L_{sd} \big/ \left(x_d - t_s^{dep}\right)$. Consequently, the total bunker cost f^{bun} of the ship over the long-haul voyage can be calculated as

$$\begin{aligned} f^{bun} &= P^{bun} \left[\alpha_{os} \left(\frac{L_{os}}{x_s - T_o^{dep}} \right)^{\beta_{os}} \cdot \left(x_s - T_o^{dep}\right) + \alpha_{sd} \left(\frac{L_{sd}}{x_d - t_s^{dep}} \right)^{\beta_{sd}} \cdot \left(x_d - t_s^{dep}\right) \right] \\ &= P^{bun} \alpha_{os} \left(L_{os}\right)^{\beta_{os}} \cdot \left(x_s - T_o^{dep}\right)^{1-\beta_{os}} + P^{bun} \alpha_{sd} \left(L_{sd}\right)^{\beta_{sd}} \cdot \left(x_d - t_s^{dep}\right)^{1-\beta_{sd}} \end{aligned} \quad (13)$$

where P^{bun} is the bunker price.

Based on the above derivation, the optimal sailing schedule problem of a containership over a long-haul voyage via Suez can be formulated as a mixed-integer nonlinear programming (MINLP) model minimizing the sum of bunker fuel cost and transit due:

$$\text{[SUEZ-MINLP]} \quad \begin{aligned} \min \quad f = f^{bun} + f^{tra} = {} & P^{bun}\alpha_{os}\left(L_{os}\right)^{\beta_{os}} \cdot \left(x_s - T_o^{dep}\right)^{1-\beta_{os}} \\ & + P^{bun}\alpha_{sd}\left(L_{sd}\right)^{\beta_{sd}} \cdot \left(x_d - t_s^{dep}\right)^{1-\beta_{sd}} \\ & + E^{sdr}\left[\begin{aligned} & D^{nor} + \min\left(0.10D^{nor}, 25000\right)\cdot y_1 \\ & \quad + \min\left(0.12D^{nor}, 30000\right)\cdot y_2 \\ & \quad + \min\left(0.05D^{nor}, 12500\right)\cdot\left(1 - y_1 - y_2 - y_3\right) \end{aligned}\right] \end{aligned} \tag{14}$$

subject to constraints: (1)-(6), (8)-(10) and

$$\underline{V} \le \frac{L_{os}}{x_s - T_o^{dep}} \le \overline{V} \tag{15}$$

$$\underline{V} \le \frac{L_{sd}}{x_d - t_s^{dep}} \le \overline{V} \tag{16}$$

$$\underline{T}_d^{arr} \le x_d \le \overline{T}_d^{arr} \tag{17}$$

In this formulation, constraints (15) and (16) impose that the sailing speeds should fall in a technically available interval. Constraint (17) ensures the predefined tactical schedule at its destination.

One may argue the calculation of waiting time in Eq. (7) and of transit due in Eq. (11) by proposing an extreme case: the ship arrives at Suez at a time between 23:00 and 04:00, e.g. 02:00 hr, but waits at the anchorage ground for next day's convoy instead of transiting the canal, in order to save the surcharge. The following proposition eliminates the possibility of the occurrence of this case.

Proposition 1. In an optimal sailing schedule, the containership arriving at Suez between 23:00 and 04:00 will not wait at the anchorage area for the next northbound convoy on the second day, but rather directly transit the canal by joining the upcoming convoy.

Proof: see Appendix B.

[SUEZ-MINLP] is a mixed-integer nonlinear programming problem in which constraint (1) contains two "mod" operators for clock time calculation and objective (14) includes the power function for bunker fuel consumption calculation. We next reformulate this MINLP model to a mixed-integer second order cone (MISOCP) model by introducing auxiliary variables. Note that a MISOCP model could be solved efficiently by CPLEX. This reformulation for one thing makes the model solvable via off-the-shelf optimization solvers, and for another provides the convenience for the model to be integrated into a more comprehensive schedule design model, such as the ship schedule recovery model proposed by Brouer et al. (2013).

3.2 A mixed-integer second order cone programming model

(i) Linearization of constraint (1)

It is straightforward to have the following proposition:

Proposition 2. If the remainder of a number A divided by another number N is R, namely,

$$A = N \times Q + R = N \times \left\lfloor \frac{A}{N} \right\rfloor + R$$

where Q is the quotient, then this remainder can be equivalently expressed by

$$R = A - N \times Q$$

$$Q \le \frac{A}{N} < Q + 1$$

$$Q \in Z^{+}. \qquad \square$$

According to Proposition 2, constraint (1) can be rewritten as follows by additionally introducing variables r, z_1 and z_2:

$$t_s^{arr-clock} = T_o^{dep-clock} + r - 24 \times z_1 \tag{18}$$

$$z_1 \le \frac{T_o^{dep-clock} + r}{24} < z_1 + 1 \tag{19}$$

$$r = \left(x_s - T_o^{dep} \right) - 24 \times z_2 \tag{20}$$

$$z_2 \le \frac{x_s - T_o^{dep}}{24} < z_2 + 1 \tag{21}$$

$$z_1, z_2 \in Z^{+} \tag{22}$$

(ii) SOCP reformulation of the objective function shown in Eq. (14)

SOCP is a state-of-the-art technique in the field of mathematical programming and it is widely used to treat a variety of nonlinearity in mathematical programming models

(Alizadeh & Goldfarb, 2003; Benson & Saglam, 2013). The high performance of SOCP commercial solvers such as CPLEX and MOSEK further strongly promotes its applications in recent years. For example, Du et al. (2011) employ SOCP to deal with the nonlinearity involved in bunker fuel calculation when solving a berth allocation problem considering bunker fuel consumption and ship emissions. Here we also adopt the same reformulation scheme.

First, two auxiliary positive variables w_1 and w_2 are introduced to substitute for the two nonlinear terms in objective (14), $\left(x_s - T_o^{dep}\right)^{1-\beta_{os}}$ and $\left(x_d - t_s^{dep}\right)^{1-\beta_{sd}}$, respectively. It is straightforward to show that model [SUEZ-MINLP] is equivalent to the following model:

$$
\begin{aligned}
\min \quad f = {} & P^{bun}\alpha_{os}\left(L_{os}\right)^{\beta_{os}} \cdot w_1 + P^{bun}\alpha_{sd}\left(L_{sd}\right)^{\beta_{sd}} \cdot w_2 \\
& + E^{sdr}\left[\begin{aligned} & D^{nor} + \min\left(0.10D^{nor}, 25000\right)\cdot y_1 \\ & \quad + \min\left(0.12D^{nor}, 30000\right)\cdot y_2 \\ & \quad + \min\left(0.05D^{nor}, 12500\right)\cdot\left(1 - y_1 - y_2 - y_3\right) \end{aligned}\right]
\end{aligned} \tag{23}
$$

subject to: (2)-(6), (8)-(10), (15)-(22) and

$$\left(x_s - T_o^{dep}\right)^{1-\beta_{os}} \le w_1 \tag{24}$$

$$\left(x_d - t_s^{dep}\right)^{1-\beta_{sd}} \le w_2 \tag{25}$$

Based on the theory on SOCP (Lobo et al., 1998), Eqs. (24) and (25) can be cast as a number of SOCP constraints provided that β_{os} and β_{sd} are greater than 1 and can be expressed as the ratio of two positive integers. In other words, constraints (24) and (25) can be equivalently formulated by the SOCP constraints below:

$$SOCP_i\left(x_s, w_1, u_{11}, u_{12}, \cdots, u_{1M_{os}}\right) \le 0, i = 1, 2, \cdots, N_{os} \tag{26}$$

$$SOCP_j\left(x_d, t_s^{dep}, w_2, u_{21}, u_{22}, \cdots, u_{2M_{sd}}\right) \le 0, \; j = 1, 2, \cdots, N_{sd} \tag{27}$$

$$u_{11}, u_{12}, \cdots, u_{1M_{os}} \ge 0, \quad u_{21}, u_{22}, \cdots, u_{2M_{sd}} \ge 0, \quad w_1, w_2 > 0 \tag{28}$$

where $\left\{u_{11}, u_{12}, \cdots, u_{1M_{os}}\right\}$ and $\left\{u_{21}, u_{22}, \cdots, u_{2M_{sd}}\right\}$ are two sets of auxiliary decision variables related to Eqs. (24) and (25) respectively. The numbers of auxiliary decision variables (M_{os} and M_{sd}) and the numbers of SOCP constraints (N_{os} and N_{sd}) depend on the parameters β_{os} and β_{sd}.

Given parameters β_{os} and β_{sd}, we can easily specify the above SOCP constraints by means of the method illustrated by Lobo et al. (1998). Here, we present the equivalent SOCP constraints to Eqs. (24) and (25) when $\beta_{os} = 2.7$ and $\beta_{sd} = 2.5$, since these will be used in our case study on a 13000-TEU containership. Now we use constraint (24) with $\beta_{os} = 2.7$ as an example to demonstrate the transformation process.

Example. Consider constraint (24) with $\beta_{os} = 2.7$.

$$\left(x_s - T_o^{dep}\right)^{1-\beta_{os}} = \left(x_s - T_o^{dep}\right)^{1-2.7} = \left(x_s - T_o^{dep}\right)^{-\frac{17}{10}} \le w_1$$

which is equivalent to

$$1 \le \left(x_s - T_o^{dep}\right)^{17} \left(w_1\right)^{10} \tag{29}$$

Since $x_s - T_o^{dep} > 0$ and $w_1 > 0$, Eq. (29) can be equivalently represented by a group of hyperbolic inequalities:

$$\begin{aligned} &\left(u_{11}\right)^2 \le \left(x_s - T_o^{dep}\right) \cdot 1, \quad \left(u_{12}\right)^2 \le u_{11} \cdot w_1, \quad \left(u_{13}\right)^2 \le u_{12} \cdot 1 \\ &\left(u_{14}\right)^2 \le u_{13} \cdot w_1, \quad 1^2 \le \left(x_s - T_o^{dep}\right) \cdot u_{14}, \quad u_{11}, u_{12}, u_{13}, u_{14} \ge 0, \quad w_1 > 0 \end{aligned} \tag{30}$$

Based on the fact that any hyperbolic inequality of the form $\left(u_1\right)^2 \le u_2 \cdot u_3, \quad u_1, u_2, u_3 \ge 0$ has an equivalent SOCP form:

$$\left\| \left(2u_1, u_2 - u_3\right) \right\|_2 \le u_2 + u_3, \quad u_1, u_2, u_3 \ge 0$$

where $\|\cdot\|_2$ is the standard Euclidean norm, Eqs. (30) can be equivalently cast into a group of SOCP constraints:

$$\begin{aligned} &\left\| \left(2u_{11}, x_s - T_o^{dep} - 1\right) \right\|_2 \le x_s - T_o^{dep} + 1, \quad \left\| \left(2u_{12}, u_{11} - w_1\right) \right\|_2 \le u_{11} + w_1 \\ &\left\| \left(2u_{13}, u_{12} - 1\right) \right\|_2 \le u_{12} + 1, \quad \left\| \left(2u_{14}, u_{13} - w_1\right) \right\|_2 \le u_{13} + w_1 \\ &\left\| \left(2, x_s - T_o^{dep} - u_{14}\right) \right\|_2 \le x_s - T_o^{dep} + u_{14}, \quad u_{11}, u_{12}, u_{13}, u_{14} \ge 0, w_1 > 0 \end{aligned} \tag{31}$$

Similarly, constraint (25) with $\beta_{sd} = 2.5$ can be equivalently represented by a group of SOCP constraints:

$$\begin{aligned} &\left\| \left(2u_{21}, x_d - t_s^{dep} - 1 \right) \right\|_2 \le x_d - t_s^{dep} + 1, \quad \left\| \left(2u_{22}, u_{21} - w_2 \right) \right\|_2 \le u_{21} + w_2 \\ &\left\| \left(2, u_{21} - u_{22} \right) \right\|_2 \le u_{21} + u_{22}, \quad u_{21}, u_{22} \ge 0, \quad w_2 > 0 \end{aligned} \tag{32}$$

Now we finally reformulate model [SUEZ-MINLP] into the MISOCP model:

[**SUEZ-MISOCP**] Objective: (23)

subject to: (2)-(6), (8)-(10), (15)-(22), (26)-(28)

4. Case study

We now conduct a case study for the impact analysis of ship traffic convoy system and tolling strategy of the Suez Canal on the optimal sailing schedule of a liner containership over a long-haul voyage, in order to answer the three questions (Q1)-(Q3) raised in Section 2.

This case study assumes that a 13000-TEU containership running on the long-haul voyage from Singapore port to Le Havre port of the LP4 service shown in Fig. 1. The nautical distance from Singapore to Suez is 5020 n miles, and the distance from Suez to Le Havre is 3130 n miles. According to the published schedule by APL (departure time from Ningbo is zero), the containership leaves Singapore on the 10th day, sails for 21 days over the Asia-Europe long-haul voyage via Suez and arrives at Le Havre on the 31st day. The sailing speed of this large containership is between 10 and 23 knots. For the coefficients in fuel calculation (MT/day), regression results based on real shipping log data suggest: $\alpha_{os} = 0.04106$, $\beta_{os} = 2.7$ (Singapore to Suez, $R^2 = 0.96$); $\alpha_{sd} = 0.07731$, $\beta_{sd} = 2.5$ (Suez to Le Havre, $R^2 = 0.94$). The normal transit due of this ship is 422175 SDR. The exchange rate from SDR to USD is chosen as $E^{sdr} = 1.41$. Bunker price is set as $P^{bun} = 300$ USD/MT without special indication.

Model [SUEZ-MISOCP] is coded by YALMIP (Löfberg, 2004) in MATLAB and solved by IBM ILOG CPLEX 12.7.1. All the experiments are run on a personal computer with Intel Core 2.60 GHz CPU and 16GB of RAM.

4.1 Impact of the Suez Canal traffic system on the optimal sailing schedule and operating cost

To quantitatively assess the impact of additionally considering the Suez Canal traffic system in schedule/speed optimization, we consider the entire long-haul voyage from Singapore to Le Havre, conduct numerical tests with different departure times (and different departure Suez-clock times) from Singapore (see the first two columns in Table 2), and collect the experimental results in Table 2. The "Benchmark" model is a traditional sailing schedule optimization model assuming that a ship could begin its transit at Suez at any clock time without consideration of the convoy system and that no surcharge would be claimed in any transit situation.

Table 2. Optimal cost structure with/without concern about the traffic system of Suez Canal (Bunker price: 300 USD/MT)

T_o^{dep} [a]	$T_o^{dep-clock}$ [b]	Benchmark [c]				Model [SUEZ-MISOCP] [e]			
		Waiting time [d]	Bunker cost	Transit due [d]	Total cost	Waiting time	Bunker cost	Transit due	Total cost
224	2	0	485806	**637567**	1123372	5	493738	595267	**1089004**
228	6	23	492084	595267	1087351	5	500337	595267	1095604
232	10	21	498494	595267	1093761	5	507174	595267	1102441
236	14	19	505044	595267	1100311	5	514261	595267	1109528
240	18	18	511731	595267	1106998	5	521610	595267	1116877
244	22	16	518564	595267	1113831	5	529236	595267	1124503
248	2	15	525546	595267	1120812	5	537152	595267	1132418
252	6	13	532682	595267	1127949	5	544489	595267	1139756
256	10	11	539977	595267	1135243	5	551326	595267	1146593
260	14	10	547433	595267	1142700	5	558413	595267	1153680

Note: Unit for time: hour. Unit for cost: USD. [a] The departure time from Ningbo Port is regarded as zero, as per the shipping schedule of service LP4 published by APL; [b] clock time at Suez; [c] CPU time for each instance is less than 1 s; [d] calculated by the optimal arrival times worked out by the benchmark model, although the benchmark model assumes no waiting and no surcharge at Suez; [e] CPU time for each instance is less than 1 s.

Experimental results in Table 2 reveal several insights: operational schedule design without consideration of the traffic convoy system at Suez may lead to infeasible schedules. This can be seen from the fact reflected by the third column of Table 2 that the assumption of zero waiting time before transit at Suez made by the "Benchmark" model often cannot be satisfied in practice. Second, an optimal sailing schedule ignoring the Suez Canal traffic system may underestimate the operating cost, even the bunker fuel cost, of the ship over the long-haul voyage. This can be mathematically explained by the fact that additionally considering the Suez Canal traffic system will bring more constraints and shrink the feasible domain of the model. Third, operational schedule optimization considering the Suez Canal convoy system ([SUEZ-MINLP] or [SUEZ-MISOCP]) might help to reduce the total operating cost of the ship over the long-haul voyage, since the transit due at Suez represents a large portion of the total cost (more than 50% at the bunker price of 300 USD/MT, and 36-40% at the bunker price of 600 USD/MT).

Note that all the instances of model [SUEZ-MISOCP] shown in Table 2 can be solved to optimality within 1 second, which shows the high computational performance of our MISOCP model and its potential to be integrated into a more comprehensive schedule/speed optimization model.

4.2 Optimal recovery plans and cost structure in different delay situations

Consider a situation in which this 13000-TEU containership is at a waypoint 500 n miles away from (before) Suez but experienced sailing delay due to bad weather/sea conditions, requiring a recovery plan to catch up the predefined schedule at Le Havre. We conduct experiments at different delay levels shown in the first column of Table 3 and report the results in Table 3 and Fig 4.

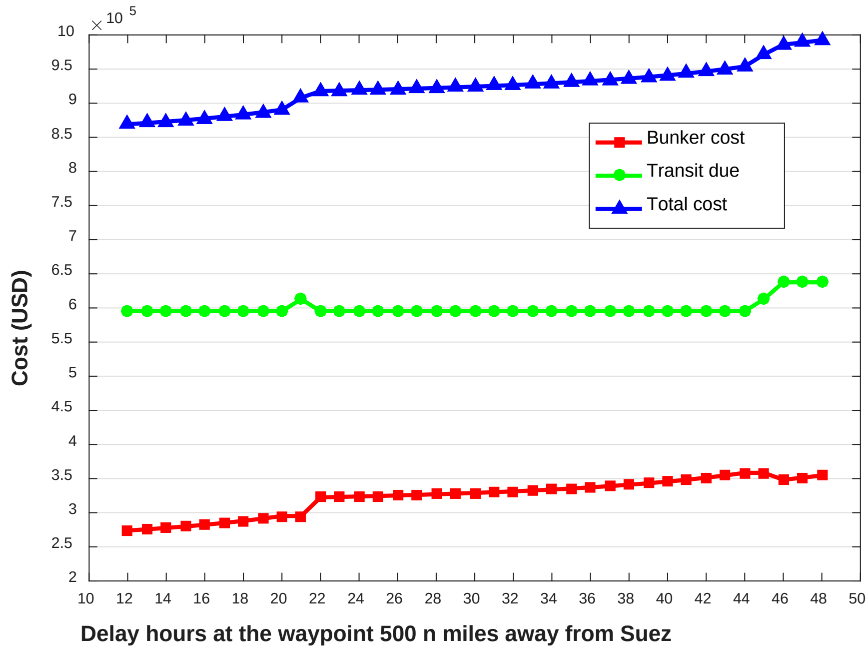

Fig 4. Cost variation against different delay levels at the waypoint 500 n miles away from Suez

Table 3. Influence of schedule delay on optimal speeds and operating cost (bunker price: 300 USD/MT)

Delay hours [a]	Speed on leg1	Speed on leg 2	Bunker cost	Transit due	Total cost
12	16.7	18.6	273895	595267	869162
13	17.2	18.6	275713	595267	870980
14	17.9	18.6	277709	595267	872976
15	18.5	18.6	279907	595267	875174
16	19.2	18.6	282337	595267	877603
17	20.0	18.6	285032	595267	880299
18	20.8	18.6	288035	595267	883302
19	21.7	18.6	291396	595267	886663

20	22.7	18.6	295176	595267	890442
21	22.7	18.6	295177	**612892**	908068
22	11.4	21.7	322507	595267	917773
23	11.6	21.7	323144	595267	918410
24	11.9	21.7	323822	595267	919089
25	12.2	21.7	324545	595267	919812
26	12.5	21.7	325318	595267	920585
27	12.8	21.7	326145	595267	921411
28	13.2	21.7	327030	595267	922297
29	13.5	21.7	327982	595267	923248
30	13.9	21.7	329005	595267	924271
31	14.3	21.7	330108	595267	925374
32	14.7	21.7	331299	595267	926566
33	15.2	21.7	332589	595267	927856
34	15.6	21.7	333989	595267	929255
35	16.1	21.7	335512	595267	930779
36	16.7	21.7	337174	595267	932440
37	17.2	21.7	338992	595267	934259
38	17.9	21.7	340988	595267	936255
39	18.5	21.7	343186	595267	938453
40	19.2	21.7	345615	595267	940882
41	20.0	21.7	348311	595267	943577
42	20.8	21.7	351314	595267	946580
43	21.7	21.7	354674	595267	949941
44	22.7	21.7	358454	595267	953721
45	22.7	21.7	358455	**612892**	971347
46	20.0	21.7	348311	**637567**	985877
47	20.8	21.7	351314	**637567**	988880
48	21.7	21.7	354674	**637567**	992241

Note: Unit for speed: knots. Unit for cost: USD. [a] Delay hours relative to predefined schedule at the waypoint 500 n miles away from Suez; the departure Suez-clock time from this waypoint can be calculated based on the data in the first two columns of Table 2.

It can be seen from Table 3 that in some delay scenarios, the ship may transit the Suez Canal at the price of high surcharge on transit due in order to avoid high speed and tremendous increase of bunker fuel consumption from Suez to Le Havre. Moreover, the occurrence of this behaviour depends both on the severity of delays and on the Suez-clock time when the ship arrives at (departs from) the waypoint. Fig. 4 reveals that the bunker cost dominates the increasing trend of the total cost caused by shipping delays, while the transit due also causes the sharp increases of the total cost at some critical points (with delay hours: 21, 45). For instance, when the delay reaches 45 hours and higher, the total cost increases significantly. In shipping practice, the shipping line should control the delay to a level lower than that represented by these critical points (21 and 45 hours) as possible as it can.

4.3 Sensitivity analysis on bunker price fluctuation and transit due adjustment

We re-conduct the experiments in Section 4.2 at different bunker prices from 300 to 600 USD/MT and show the cost structure in optimal solutions in Fig. 5. Meanwhile, to imitate the behaviour of SCA on transit due adjustment, we also re-conduct the experiments in Section 4.2 by keeping the bunker price at 300 USD/MT but letting the normal transit due of this ship at Suez perturb at most $\pm 20\%$ from its current value 422175 SDR, and plot the results in Fig. 6.

Fig. 5 shows that bunker price does influence the banker cost and thus the total cost. When the bunker price is low and the transit due represents a larger portion of the total cost, the ship tends to be averse to the risk of higher transit surcharge. When the bunker price is high, incurring high transit surcharge might help to reduce the bunker cost from Suez to Le Havre and thus the total cost. Generally, the optimal recovery plan is mainly determined by the predefined tactical schedule, delay situation and clock time at Suez, but not pretty much affected by bunker price.

Experimental results in Fig. 6 are analyzed as follows. Although the adjustment of transit due obviously influences the total cost incurred of the ship, the optimal sailing schedule obtained from our model demonstrates its robustness against the variation of transit due: $\pm 20\%$ adjustment of the transit due by SCA will not change the optimal speeds adopted for schedule recovery, which explains the upper subplot of Fig. 6. This again shows the fact that the optimal schedule recovery plan is mainly determined by the predefined tactical schedule, delay situation and Suez-clock time.

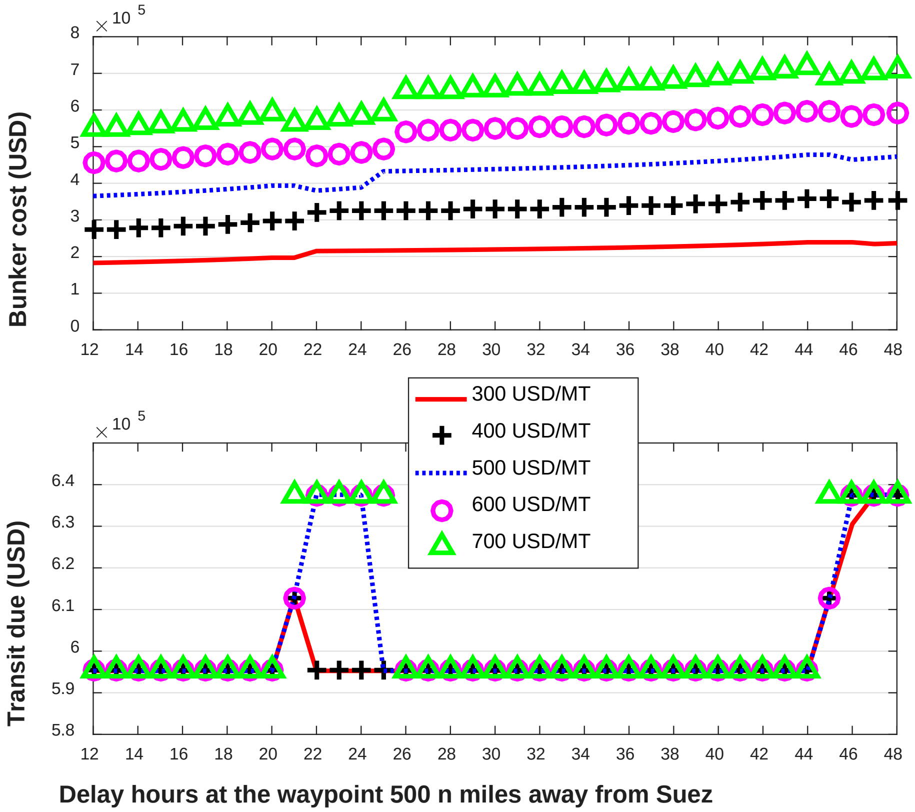

Fig 5. Influence of bunker price on operating cost at different delay levels

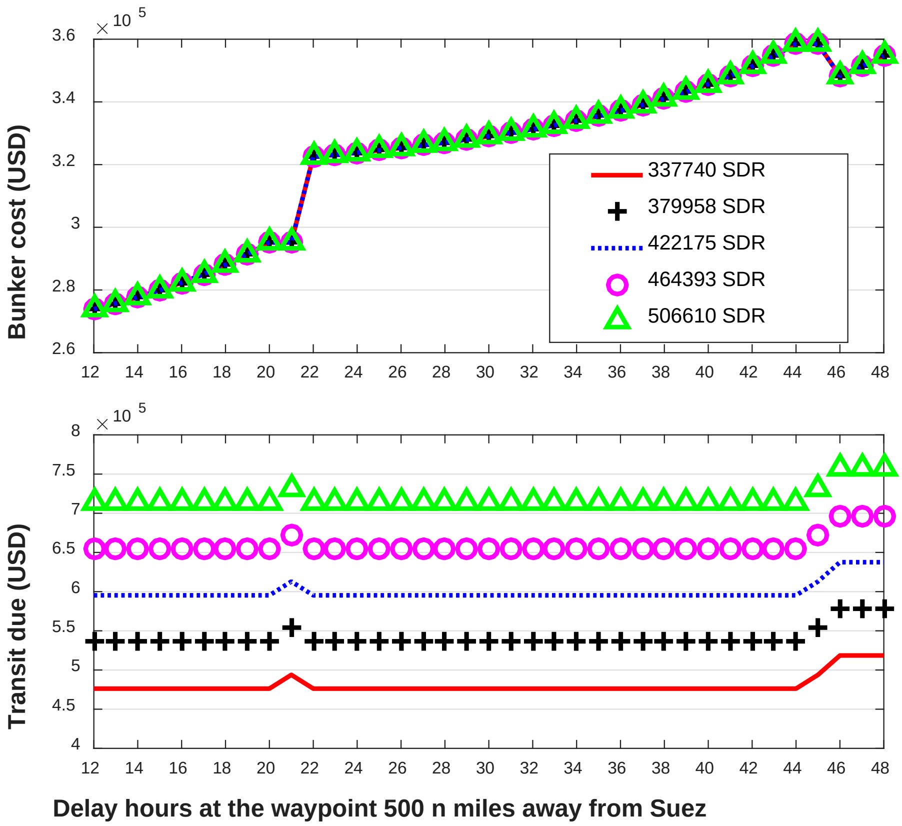

Fig 6. Influence of transit due adjustment on operating cost at different delay levels

5. Conclusions

This paper carries out a quantitative impact analysis of traffic regulations of the Suez Canal and its toll pricing policy on the optimal sailing schedule of a containership over a long-haul voyage. Based on the ship convoy system at Suez and the stepwise structure of transit due function, we set up a MINLP model for operational schedule optimization of a containership over a long-haul voyage via Suez. To take advantage of off-the-shelf optimization solvers, we linearize the modulo operators in clock-time calculation, treat the nonlinearity in forms of power functions with the state-of-the-art SOCP technique, and finally obtain a MISOCP formulation, whose high computational performance supports its industrial application and possible extension to a more comprehensive operational schedule optimization model. At last, we perform a case study on a 13000-TEU containership running on the LP4 service operated by APL. Experimental results based on the real data collected from a global container shipping line well answer some interesting questions regarding this novel problem.

This paper makes the first move to address the impact of traffic convoy system and toll pricing policy at Suez on ship schedule (speed) optimization. We highlight some basic managerial insights obtained through numerical experiments which constitute the main contribution of this paper: (i) ignoring the influence of the traffic system at Suez in ship schedule optimization may lead to infeasible sailing schedules, and underestimate the operating cost (even the bunker cost) of a containership on a long-haul voyage via Suez; (ii) our model helps to work out an optimal sailing schedule by jointly considering bunker cost and transit due at Suez; (iii) the optimal recovery plan to treat sailing delay is mainly determined by the predefined tactical schedule, delay situation and Suez-clock time, but not pretty much affected by the levels of bunker price and transit due. The proposed model can be used by a shipping line to determine the operational-level schedule of a containership over a long-haul voyage via Suez, and to make the operational-level decision on route choice (Suez versus the Cape of Good Hope).

The model in this study is based on the traffic convoy system and toll pricing policy of the New Suez Canal opening in August 2015 after a one-year dredging work between August 2014 and July 2015. We also conducted a similar study based on the traffic convoy system and toll pricing policy of the old Suez Canal, and found that the old

Suez Canal (northbound transit) needs a minor change towards the mathematical model due to the necessity of additionally introducing a binary variable y_4 (i.e. $\{y_1, y_2, y_3, y_4\}$ associated with the variable $t_s^{arr-clock}$). However, the basic managerial insights obtained through numerical experiments remain the same.

Future studies can investigate several relevant issues. First, a cost minimization model can be set up for the optimal sailing schedule of a containership over a long-haul voyage with southbound transit through the Suez Canal. Second, the proposed model in this paper could be plugged into a larger model which considers more sailing legs, port calls and more complicated cost structures. Third, integrating the influence of weather/sea conditions into schedule/speed optimization of a containership over a long-haul voyage via Suez is another challenging issue.

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Appendix A. Notations

Input parameters

D^{nor}	The normal transit due of the ship at Suez charged by SCA (unit: SDR)
E^{sdr}	The exchange rate from SDR to USD
L_{os}	The distance from the ship's current position o to the Suez Canal (n miles)
L_{sd}	The distance from Suez to the ship's the destination d (n miles)
M	A sufficiently large constant
P^{bun}	The bunker price (USD)
$\left[\underline{T}_d^{arr}, \overline{T}_d^{arr}\right]$	The predefined arrival time window of the ship at the destination d
T_o^{dep}	The time (point) when the ship departs from its current position o (the beginning time of the whole voyage is regarded as time zero)

$T_o^{dep-clock}$	The Suez clock time corresponding to T_o^{dep}
T_s^{tra}	The time (duration) needed for the ship to transit the Suez Canal
$\left[\underline{V}, \bar{V}\right]$	The sailing speed range of the ship (knots)
α_{os}, β_{os}	The two coefficients in bunker fuel calculation for the leg from origin o to Suez
α_{sd}, β_{sd}	The two coefficients in bunker fuel calculation for the leg from Suez to the destination d

Decision variables

x_d	The arrival time (point) of the ship at its destination d
x_s	The arrival time (point) of the ship at Suez

Auxiliary variables

f	The total cost of the ship over the long-haul voyage (USD)
f^{bun}	The bunker fuel cost of the ship over the long-haul voyage (USD)
f^{tra}	The transit due of the ship charged by SCA, including the normal transit due and the surcharge (if any), in terms of USD
$t_s^{arr-clock}$	The Suez clock time corresponding to x_s
t_s^{dep}	The departure time (point) of the ship from Suez
t_s^{wait}	The waiting time (duration) of the ship at Suez before transit (hours)
v_{os}	The sailing speed of the ship from current position o to the Suez Canal (knots)
v_{sd}	The sailing speed of the ship from Suez to the destination d
y_1, y_2, y_3	Binary variables indicating associated with the variable $t_s^{arr-clock}$

Appendix B. Proof of Proposition 1

Proof. We apply the proof by contradiction. Assume that the containership arrives at Suez between 23:00 and 04:00 and chooses to wait for next day's convoy by following an optimal schedule $\pi = \left(x_s(\pi), t_s^{arr-clock}(\pi), t_s^{dep}(\pi), x_d(\pi)\right)$, where the four elements representing the schedule respectively denote the values of x_s, $t_s^{arr-clock}$, t_s^{dep} and x_d associated with schedule π.

If the optimal sailing speed to Suez $v_{os}(\pi) = L_{os} \big/ \left(x_s(\pi) - T_o^{dep} \right) > \underline{V}$, then the ship can further reduce its sailing speed and arrive at Suez at a time $x_s(\sigma)$ later than $x_s(\pi)$ but before the next 23:00, which forms a new schedule

$$\sigma = \left(x_s(\sigma), t_s^{arr-clock}(\sigma), t_s^{dep}(\sigma), x_d(\sigma) \right)$$

where

$$x_s(\sigma) > x_s(\pi) \tag{A1}$$

$$t_s^{arr-clock}(\pi) + \left(x_s(\sigma) - x_s(\pi) \right) \le 47 \tag{A2}$$

$$t_s^{arr-clock}(\sigma) = \left[t_s^{arr-clock}(\pi) + \left(x_s(\sigma) - x_s(\pi) \right) \right] \bmod 24 \le 23 \tag{A3}$$

$$t_s^{dep}(\sigma) = t_s^{dep}(\pi) \tag{A4}$$

$$x_d(\sigma) = x_d(\pi) \tag{A5}$$

Based on the domain knowledge $\alpha_{os}, \alpha_{sd} > 0, \beta_{os}, \beta_{sd} \in [2.5, 3.5)$, we can compare the cost of these two schedules:

$$f^{tra}(\sigma) = f^{tra}(\pi) = E^{sdr} \cdot D^{nor} \tag{A6}$$

$$\begin{aligned} f^{bun}(\sigma) &= P^{bun} \left[\alpha_{os} (L_{os})^{\beta_{os}} \cdot \left(x_s(\sigma) - T_o^{dep} \right)^{1-\beta_{os}} + \alpha_{sd} (L_{sd})^{\beta_{sd}} \cdot \left(x_d(\sigma) - t_s^{dep}(\sigma) \right)^{1-\beta_{sd}} \right] \\ &= P^{bun} \left[\alpha_{os} (L_{os})^{\beta_{os}} \cdot \left(x_s(\sigma) - T_o^{dep} \right)^{1-\beta_{os}} + \alpha_{sd} (L_{sd})^{\beta_{sd}} \cdot \left(x_d(\pi) - t_s^{dep}(\pi) \right)^{1-\beta_{sd}} \right] \\ &< P^{bun} \left[\alpha_{os} (L_{os})^{\beta_{os}} \cdot \left(x_s(\pi) - T_o^{dep} \right)^{1-\beta_{os}} + \alpha_{sd} (L_{sd})^{\beta_{sd}} \cdot \left(x_d(\pi) - t_s^{dep}(\pi) \right)^{1-\beta_{sd}} \right] \\ &= f^{bun}(\pi) \end{aligned} \tag{A7}$$

Eqs. (A6) and (A7) indicate the fact that the slow-steaming strategy suggested by schedule σ will decrease the bunker fuel consumption over the first leg (from origin to Suez), without changing the transit due and bunker fuel consumption over the second leg from Suez to destination. This contradicts the assumption that schedule π is optimal.

If the optimal sailing speed to Suez $v_{os}\left(\pi\right)=\underline{V}$, then the ship can adopt a new sailing schedule λ by speeding up a little bit to arrive at Suez at 23:00 and catching the upcoming convoy without paying any surcharge ($f^{tra}\left(\lambda\right)=f^{tra}\left(\pi\right)=E^{sdr}\cdot D^{nor}$), which will increase the sailing time over the second leg from Suez to destination by 24 hours:

$$\lambda=\left(x_s\left(\lambda\right),t_s^{arr-clock}\left(\lambda\right),t_s^{dep}\left(\lambda\right),x_d\left(\lambda\right)\right)$$

where

$$0<x_s\left(\pi\right)-x_s\left(\lambda\right)<5 \tag{A8}$$

$$t_s^{arr-clock}\left(\lambda\right)=\left[t_s^{arr-clock}\left(\pi\right)-\left(x_s\left(\pi\right)-x_s\left(\lambda\right)\right)\right]\bmod 24=23 \tag{A9}$$

$$t_s^{dep}\left(\lambda\right)=t_s^{dep}\left(\pi\right)-24 \tag{A10}$$

$$x_d\left(\lambda\right)=x_d\left(\pi\right) \tag{A11}$$

Without loss of generality, the following reasonable assumptions are made based on the reality of long haul sailing via Suez:

$$v_{sd}\left(\pi\right)>\frac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}-5}\approx\underline{V}=\frac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}} \tag{A12}$$

$$x_s\left(\pi\right)-T_o^{dep}\geq 24 \tag{A13}$$

$$x_d\left(\pi\right)-t_s^{dep}\left(\pi\right)\geq 6\times 24=144 \tag{A14}$$

Compared to schedule π, the ship decreases its sailing time over the first leg from origin to Suez by at most 5 hours if it adopts schedule λ. We thus have:

$$f_{os}^{bun}\left(\lambda\right)\leq P^{bun}\alpha_{os}\left(L_{os}\right)^{\beta_{os}}\cdot\left(x_s\left(\pi\right)-T_o^{dep}-5\right)^{1-\beta_{os}} \tag{A15}$$

The Taylor expansion of the right-hand side of Eq. (A15) is:

$$P^{bun}\alpha_{os}\left(L_{os}\right)^{\beta_{os}}\cdot\left(x_s\left(\pi\right)-T_o^{dep}-5\right)^{1-\beta_{os}}$$

$$=P^{bun}\begin{bmatrix}\alpha_{os}\left(L_{os}\right)^{\beta_{os}}\cdot\left(x_s\left(\pi\right)-T_o^{dep}\right)^{1-\beta_{os}}+\alpha_{os}\left(L_{os}\right)^{\beta_{os}}\left(\beta_{os}-1\right)\left(x_s\left(\pi\right)-T_o^{dep}\right)^{-\beta_{os}}\cdot 5\\ +\alpha_{os}\left(L_{os}\right)^{\beta_{os}}\left(\beta_{os}-1\right)\beta_{os}\left(x_s\left(\pi\right)-T_o^{dep}-\theta\right)^{-\beta_{os}-1}\cdot 5^2/2!\end{bmatrix}$$

$$=f_{os}^{bun}\left(\pi\right)+P^{bun}\begin{bmatrix}\alpha_{os}\left(v_{os}\left(\pi\right)\right)^{\beta_{os}}\left[5\left(\beta_{os}-1\right)\right]\\ +\alpha_{os}\left(\dfrac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}-\theta}\right)^{\beta_{os}}\cdot\dfrac{\beta_{os}\left(\beta_{os}-1\right)}{x_s\left(\pi\right)-T_o^{dep}-\theta}\cdot\dfrac{25}{2}\end{bmatrix}$$

where the third term containing θ is the remainder term of Taylor expansion and θ is between 0 and 5. Thus, the increase of bunker fuel cost of schedule λ over the first leg, compared to schedule π, can be bounded with the following derivation:

$$\begin{aligned}\Delta f_{os}^{bun}&=f_{os}^{bun}\left(\lambda\right)-f_{os}^{bun}\left(\pi\right)\\
&\le P^{bun}\left[\alpha_{os}\left(v_{os}\left(\pi\right)\right)^{\beta_{os}}\left[5\left(\beta_{os}-1\right)\right]+\alpha_{os}\left(\frac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}-\theta}\right)^{\beta_{os}}\cdot\frac{\beta_{os}\left(\beta_{os}-1\right)}{x_s\left(\pi\right)-T_o^{dep}-\theta}\cdot\frac{25}{2}\right]\\
&\le P^{bun}\left[\alpha_{os}\left(v_{os}\left(\pi\right)\right)^{\beta_{os}}\left[5\left(\beta_{os}-1\right)\right]+\alpha_{os}\left(\frac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}-5}\right)^{\beta_{os}}\cdot\frac{\beta_{os}\left(\beta_{os}-1\right)}{x_s\left(\pi\right)-T_o^{dep}-5}\cdot\frac{25}{2}\right]\\
&=P^{bun}\left[\alpha_{os}\left(\underline{V}\right)^{\beta_{os}}\left[5\left(\beta_{os}-1\right)\right]+\alpha_{os}\left(\frac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}-5}\right)^{\beta_{os}}\cdot\frac{\beta_{os}\left(\beta_{os}-1\right)}{x_s\left(\pi\right)-T_o^{dep}-5}\cdot\frac{25}{2}\right]\\
&\le P^{bun}\left[\alpha_{os}\left(\underline{V}\right)^{\beta_{os}}\left[5\left(\beta_{os}-1\right)\right]+\alpha_{os}\left(\frac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}-5}\right)^{\beta_{os}}\cdot\frac{\beta_{os}\left(\beta_{os}-1\right)}{24-5}\cdot\frac{25}{2}\right]\\
&\overset{\text{Apply }\beta_{os}<3.5}{<}P^{bun}\left[\alpha_{os}\left(\underline{V}\right)^{\beta_{os}}\left[5\left(3.5-1\right)\right]+\alpha_{os}\left(\frac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}-5}\right)^{\beta_{os}}\cdot\frac{3.5\left(3.5-1\right)}{24-5}\cdot\frac{25}{2}\right]\\
&=P^{bun}\left[\alpha_{os}\left(\underline{V}\right)^{\beta_{os}}\cdot 12.5+\alpha_{os}\left(\frac{L_{os}}{x_s\left(\pi\right)-T_o^{dep}-5}\right)^{\beta_{os}}\cdot 5.7566\right]\end{aligned}\tag{A16}$$

This equation has a clear economic meaning that the bunker fuel increase over the first leg caused by schedule λ can be upper bounded by the fuel consumption of the same ship in a 12.5-hour sailing at speed $\underline{V}$ plus that in a 5.8-hour sailing at an extremely low speed close to $\underline{V}$. Similarly, we can obtain a lower bound of the bunker cost decrease over the second leg from Suez to destination with schedule λ:

$$\Delta f_{sd}^{bun} = f_{sd}^{bun}(\pi) - f_{sd}^{bun}(\lambda) \geq P^{bun}\alpha_{sd}\left(v_{sd}(\pi)\right)^{\beta_{sd}} \cdot 18.5 \qquad \text{(A17)}$$

This bound can be interpreted as the bunker fuel cost of the ship in an 18.5-hour sailing over the second leg at the speed $v_{sd}(\pi)$. The two bounds in Eqs. (A16) and (A17) and the assumption reflected by Eq. (A12) clearly support the result below:

$$\Delta f_{os}^{bun} < \Delta f_{sd}^{bun} \qquad \text{(A18)}$$

which can be translated to the fact $f^{bun}(\lambda) < f^{bun}(\pi)$ that contradicts our assumption on the optimality of schedule π. □

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