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MODELLING: FROM ICME 5 TO PRACTICE

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Looking at the enormous number of choices within the program for ICME 5 it was daunting to try and choose Areas, Themes, and Topics which would be the most interesting and useful to attend. As with most participants I made some excellent choices and some disappointing ones. On the whole, however, the positive impressions far outnumbered the negative ones. As a person involved with first and second year university mathematics courses I decided to concentrate on some sessions which possibly could help improve my teaching and involvement with students. Particularly useful in this regard were sessions on the teaching of statistics. Here, however, I want to concentrate on several aspects of modelling which arose in the **Theme Group on "Applications and Modelling"**.

The Theme Group was organized by David Lee of the South Australian Institute of Technology, Dick Lesh from the United States, and Mogens Niss from Denmark. In the four sessions, discussions were held with participants split first into groups by educational level of involvement, second into groups by interest (e.g., purpose, curriculum issues, psychological research, resources), third into the intersection of the first two, and fourth back into educational level. This served to mix people and allow for discussion in small groups. Not always did this lead to substantive conclusions, but just listening to people from other parts of the world with their problems and/or solutions added to the experience of others. In the groups I attended there were people from Australia, New Zealand, Canada, the United States, the United Kingdom, Denmark, and Hong Kong. The human interest side came through on several occasions. I recall one of the groups being spellbound as a woman, who was tiny in stature but full of energy, described the plight of teachers in the state of Mississippi in the U.S.A.

THE MEANING OF "MODELLING"

As is often the case when poeple from diverse backgrounds come together, even such a fundamental term as "modelling" meant different things to different people. The context in which modelling was considered varied widely among the participants of one group. At one extreme modelling was equated with "application" and represented the presence of examples tacked on the end of mathematical expositions to illustrate their "usefulness". At the other extreme, modelling was seen as an end in itself, to be taught in a separate course, probably at a third-year tertiary level, after the mathematical techniques required had been developed in other courses. Between these extremes there was a great variety of experience in combining modelling and mathematical content in courses. Some people wanted to know how to convince colleagues to increase the modelling content; some wanted to know of resourses to enhance the modelling they were doing; and others wanted to know how to get started.

Having been involved for some years in teaching first year courses in "discrete modelling" and "continuous modelling" — both of which are part of three year sequences — I was interested to hear how others implemented the modelling aspect within courses which also by their nature contained mathematical content. In a third year course on modellling it may be possible to give a student one or two projects to work on for the entire course, but this is not possible at the first year level when students are learning mathematical techniques at the same time. Two suggestions which appeared to have potential for my situation came out of the discussion groups. They may also be of interest to senior secondary teachers who would like to increase the modelling component of their courses. To set the scene let me describe some aspects of one of my courses as it has operated in the past.

To increase my continuous modelling students' awareness of what modelling was about and to expose them to reading, rather than just listening in lectures, as a way of learning about mathematics, I required them to read one of the excellent little modules prepared by COMAP (Consortium for Mathematics and its Applications): *"Glottochronology: An Application of Calculus to Linguistics"*. The mathematics was easy, so the point was to see a model in a field which was new to all students. I think this is important because to use only examples from physics gives some students a definite advantage over others. The reading was popular with students *but* they were reading about modelling and not doing it themselves. To increase my students' critical thinking about the assumptions necessary in setting up a mathematical model, one assignment required them to criticise a model of a one-way social process, in this case the transition from illiteracy to literacy. Again the mathematics was easy. However to make it so, a great many simplifying assumptions had to be made. The students enjoyed tearing the model apart *but* they were criticising someone else's model and not creating their own.

Why did I not turn the students loose on their own to create models? Looking back, it was probably a lack of confidence: both in their ability to be successful and my ability to assess their results! It was David Lee who suggested that every day in an Australian newspaper it was possible to find an article which suggested a possibility for mathematical modelling. So why not try?

STUDENT ASSIGNMENT

In the first assignment after ICME 5 I asked my students to find an article in a newspaper or news magazine within the previous month which suggested that a mathematical model could be applied.

The question then arose as to whether the students would have the modelling skills necessary to set up and test a model. We had discussed the usual basic "model for modelling" shown in Figure 1: the real world supplies the information on a

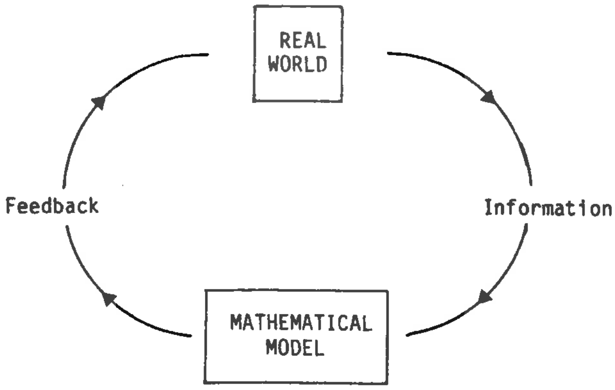

Figure 1

problem; this is then translated into a mathematical problem and solved, with the result being fed back to the real world. With some informal discussion on how a model is set up and the comment that the cycle may have to be repeated several times before the result is satisfactory, this still may be insufficient for beginning modellers. Again, however, a useful suggestion arose from ICME 5. This time it came from Dinesh Srivastava of the Correspondence School of the Education Department of Victoria. As part of his presentation he gave out a booklet associated with a short course on mathematical modelling. In it was the flow chart shown in Figure 2 for modelling real world situations. This provided a frame upon which students could build their modelling attempts.

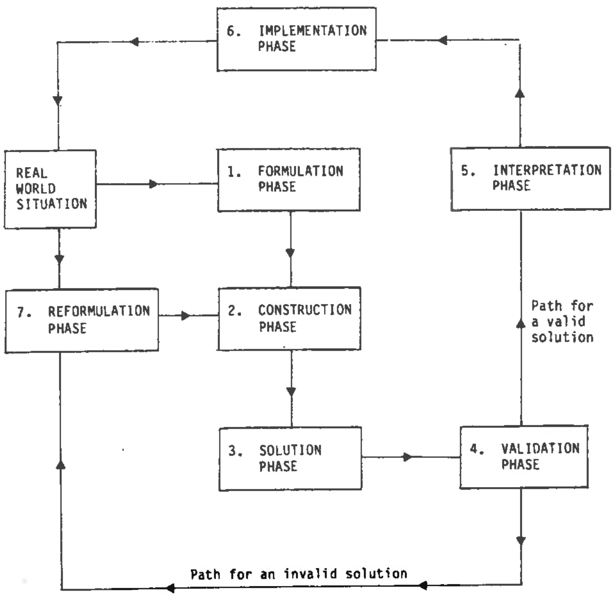

Figure 2

The assignment as it was finally constructed asked the students to use the "model for modelling" in Figure 2 to model two situations: one, for practice, which was based on anything they had met in the course thus far and one which was based on the article they had found in the news media. Fairly rigid instructions were given to limit the amount which was written as I felt that some students would get carried away with enthusiasm for the project. Students were also given the opportunity to criticise the "model for modelling" if they had difficulty using it.

RESULTS

The results of the assignment were quite interesting. About half of the students (38) attempted at least one of the modelling situations. This was about 25% less than the usual number who handed in assignments. Some students failed to follow the instructions, either writing much too much or choosing an article from a science publication rather than the news media. Some of these efforts were very good, nevertheless, and reflected considerable effort on the part of students.

Of the situations encountered previously within the course, perhaps the easiest for the students to fit into the model in Figure 2 was a discussion of population models. Very briefly, several students suggested variations on the following scheme.

- REAL WORLD SITUATION: Population of a growing living species which is to be studied.
- FORMULATION PHASE: Statement of assumptions; e.g., no immigration or emigration allowed, population growth a constant proportion of current population, a continuous approximation to a discrete problem (Malthusian model).
- CONSTRUCTION PHASE: For population p at time t, $p'(t) = ap(t)$ with $p(t_0) = p_0$, $a > 0$.
- SOLUTION PHASE: $p(t) = p_0 e^{a(t-t_0)}$.
- VALIDATION PHASE: Breaks down as t becomes large.
- REFORMULATION PHASE: Assume population growth such that initially it reflects exponential growth but slows as the population becomes large, to level off at a limiting value (Logistic model).
- CONSTRUCTION PHASE: For population p at time t, $p'(t) = (a-bp(t))p(t)$ with $p(t_0) = p_0$, $a > 0$, $b > 0$, $b < a$.
- SOLUTION PHASE: $p(t) = ap_0/(bp_0 + (a-bp_0)e^{-a(t-t_0)})$.
- VALIDATION PHASE: $\lim p(t) = a/b$, so model does not break down as t *becomes large.*
- INTERPRETATION PHASE: Examination of implications of various values of a and b for the problem.
- IMPLEMENTATION PHASE: Using actual data (e.g., U.S. census) and values of a and b, apply model to predict future population trends.
- REAL WORLD SITUATION: Make judgement about adequacy of model.
- REFORMULATION PHASE: May have to return here and complete another loop.

It could be argued that at times the problem had to be stretched to fit into the mould but at least it was illustrative of the procedure involved. Not every situation chosen included every phase and students were asked to comment when this occurred. Other situations which were discussed were the calculation of compound interest, a mixing problem, radioactive decay, the homicide problem using Newton's law of cooling, alcohol absorption and accident risk, the Van Meegeran art forgeries, and the Buffon Needle Problem. References for some of these problems are given at the end. Most students were more confident when working with a situation where "the solution" was known and could be fitted into the correct place in Figure 2.

The attempts at modelling situations which the students had discovered for themselves, however, were fascinating. First, the choice of situation showed a great deal of thought in many cases. Second, great care was taken by some students to think through the procedure needed to formulate and test a model. Several categories of articles emerged when the students' papers were collated. A few students chose articles where the use of statistics was the modelling method advocated. These included a Gallop Poll on the cost of living in Australia, a TIME poll on Reagan's lead in the U.S. presidential race, and a survey on the lifetimes of various brands of batteries. Another group of students chose articles with a medical theme, usually employing probability as the modelling device. These included assessing the success rate of the heart transplant program in Australia, picking a child's sex, inheriting diabetes, and predicting cycles of cancers in children. Often not enough information was available to complete the model and obtain "answers" in these two categories. Students did not seem to worry about this and were happy to speculate about some parts of the modelling procedure.

A third category of articles chosen showed a more open-minded attitude toward the task of modelling. The students were not looking for some recognizable reference to mathematics with which they were familiar but for situations which lent themselves to "some sort of mathematization". Among the interesting choices were the sinking of Mururoa Atoll, the poisoning of Queenslanders from fish caught in Hervey Bay, building approvals in the housing sector, and the donkey population in Western Australia. One student took an article on a weekend shopping stampede at a new Hobart garden centre and suggested a queuing model for extra staff and cash registers. Another did a fine job of developing a simple exponential model, then revising it to apply to Argentina's foreign debt problem. Although not suggesting a mathematical model himself, one other student used an article about the power costs from nuclear-and coal-fuelled power stations to illustrate what goes wrong when various phases of the modelling procedure are ignored. In this particular situation the predictions of the early 1970's, that nuclear power would be cheaper than coal power, have turned out to be incorrect in the mid-1980's.

Although some suggestions were either simplistic or unrealistic to implement, the students seemed to be aware that this was the case and commented upon it. The experience was generally enjoyed by those who put some effort into it. As the teacher I have no hesitation in recommending an exercise where students are asked to choose a situation from "current affairs" and attempt to fit it into a mathematical modelling procedure. I am somewhat less sure, however, about prescribing a detailed model as outlined in Figure 2. Unfortunately some students, even if *told* otherwise, treat a model which is typed in black-and-white on an assignment sheet as if it were the embodiment of eternal truth. If the aim is to instil a sense of flexibility and adaptability in thinking about modelling then perhaps one should be less prescriptive.

Only three students made critical comments about the suggested "model for modelling". One of these, Peter Trueman, was disturbed about the ordering of the INTERPRETAION AND IMPLEMENTATION PHASES and the fact that there was not enough contact with the REAL WORLD SITUATION throughout the modelling operation. His changes to the "model for modelling", shown in Figure 3, put the REAL WORLD SITUATION at the centre and indicate the frequent interaction of it with the phases of modelling which I had stressed in an informal way with the students. Perhaps next year I should put Peter Trueman's "model for modelling" forward for criticism by other students! Alternately, instead of suggesting a procedure to follow, perhaps the students should be asked to develop their own "models for modelling". References to other schemes for modelling are given at the end.

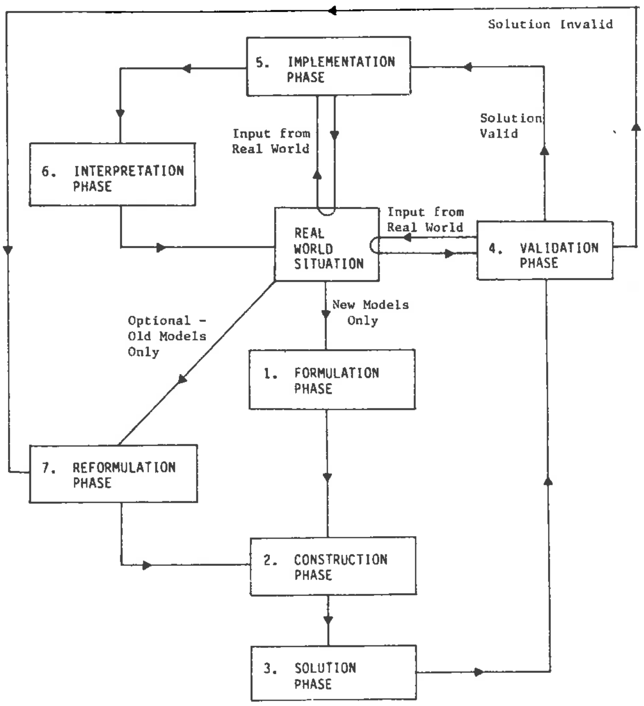

Figure 3

BENEFITS OF ICME 5

This discussion has been intended to illustrate one practical benefit of attendance at ICME 5. Further practical suggestions which I picked up at ICME still await trial. Some of the other benefits were more nebulous: listening to people like Henry Pollack, Phil Davis, and Anthony Ralston give their ideas on where mathematics and the curriculum ought to be heading provided food for thought. As well, summaries by Douglas Grouws and Allan Bell kept me in contact with the current thinking about the effects of research on the teaching of mathematics. The enormity of the program at ICME 5 certainly demonstrated that it is impossible to know everything about every facet of mathematics education. To concentrate only on one's special interest, however, would have meant missing the opportunity to ascertain trends and absorb enthusiasm at many levels and interest. I felt that this latter absorbing of enthusiasm was one of the lasting benefits of ICME 5.

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