

## **Measuring Pre-service teachers knowledge of mathematics for teaching**

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From the time that Shulman (1987) described the nature of teachers' knowledge there have been a number of attempts to identify tools to measure that knowledge. Shulman's seminal work identified seven knowledge types that contribute to teachers' knowledge: (i) content knowledge; (ii) general pedagogical knowledge; (iii) curriculum knowledge; (iv) pedagogical content knowledge (PCK); (v) knowledge of learners and their characteristics; (vi) knowledge of education contexts; and (vii) knowledge of education ends, purposes, and values (p. 8).

In the mathematics education field, this seminal work has led to considerable efforts to identify and measure teachers' knowledge beyond de facto measures of mathematics content knowledge obtained from levels of mathematics courses taken. In particular, there has been a focus on the specialised mathematics knowledge that teachers' draw on in their day-to-day work. Such knowledge goes beyond mathematics skills and understanding, and encompasses specialised aspects such as useful forms of representation, explanations and examples of the domain that can be used to help students learn mathematics. Hill, Schilling and Ball (2004) reported on initial attempts to develop measures of elementary teachers' mathematical knowledge for teaching. Callingham and Watson (2011) have described the development of measures of middle school teachers' pedagogical content knowledge in statistics.

The application of teachers' specialised mathematical knowledge is likely also to be the result of teachers' beliefs about teaching and the nature of the knowledge domain. Beswick (2011) has argued from a theoretical perspective that if beliefs are regarded as anything that is "true", then beliefs and knowledge are one and the same. Beswick, Callingham and Watson (2011) built on this idea by using Rasch modelling to identify an underlying "thick" construct of teachers' knowledge that combined several of Shulman's knowledge types.

This paper reports on the ongoing development of an instrument to measure pre-service teachers' knowledge of mathematics for teaching as part of an ALTC project. Three subscales were used, beliefs, mathematics content knowledge and mathematics pedagogical content knowledge, to create a combined scale of pre-service teachers' mathematical knowledge. Initial results suggest that the combined scale provides a way of conceptualising the development of teachers' specialised mathematics teaching knowledge.

### **Key Words**

Assessment and Measurement SIG

## INTRODUCTION

From the time that Shulman (1987) described the nature of teachers' knowledge there have been a number of attempts to identify tools to measure that knowledge. Shulman's seminal work identified seven knowledge types that contribute to teachers' knowledge: i) content knowledge; (ii) general pedagogical knowledge; (iii) curriculum knowledge; (iv) pedagogical content knowledge (PCK); (v) knowledge of learners and their characteristics; (vi) knowledge of education contexts; and (vii) knowledge of education ends, purposes, and values (p. 8).

In the mathematics education field, this seminal work has led to considerable efforts to identify and measure teachers' knowledge beyond de facto measures of mathematics content knowledge obtained from levels of mathematics courses taken. In particular, there has been a focus on the specialised mathematics knowledge that teachers' draw on in their day-to-day work. Such knowledge goes beyond mathematics skills and understanding, and encompasses specialised aspects such as useful forms of representation, explanations and examples of the domain that can be used to help students learn mathematics. Much of this interest has been driven by tests for teacher registration in the USA where there is considerable variation across the tests used for elementary teachers in mathematics (Hill, Schilling & Ball. 2004). Some tests address only mathematics content, whereas others attempt to capture the kind of mathematical knowledge implicit in being able to respond to students' questions or errors.

In Australia, there is growing interest in teachers' knowledge. The Australian Institute for Teaching and School Leadership (AITSL) recently released National Standards for Teachers covering three domains: Professional Knowledge, Professional Practice and Professional Engagement (AITSL, 2011) at four levels of competence. As yet no decision has been made concerning how these might be implemented, but the inclusion of domains broader than content knowledge indicates that some of the complexity of teachers' work is acknowledged. In Queensland a pre-registration test has been developed covering three domains across the knowledge areas. These domains and knowledge areas are summarised in Table 1. The first construct, "Required knowledge and understanding of content and processes to teach ..." covers aspects of the curriculum content defined by the local curriculum standards for Years 3, 5, 7 and 9. The second construct, "Knowledge about teaching of ..." addresses what could broadly be described as pedagogical content knowledge (PCK) and is defined as

- knowledge of curriculum frameworks,
- teaching of literacy/numeracy/science [and ]
- diagnosis, monitoring and assessment

(State of Queensland (Queensland College of Teachers), 2011).

The last construct is personal numeracy or literacy, and addresses the kinds of mathematical knowledge that might be expected of an educated teacher within the community.

Table 1

*Domains (Constructs) of Teacher Knowledge for Queensland Pre-registration Test*

	<b>Construct</b>	<b>Priority area</b>
1	<i>Required knowledge and understanding of content and processes to teach ...</i>	Literacy Numeracy Science
2	<i>Knowledge about teaching of ...</i>	Literacy Numeracy Science
3	Personal literacy and numeracy skills	Literacy Numeracy

With this growing interest, and the introduction of a high stakes test in at least one state, it is timely to consider the nature of teachers' knowledge. This paper reports on the ongoing development of an instrument to measure pre-service teachers' knowledge of mathematics for teaching as part of an ALTC project, "Building the Culture of Evidence-based Practice in Teacher Preparation for Mathematics Teaching (CEMENT)". The purpose of the project is to provide an evidence-base and instruments for institutions providing pre-service teacher education to inform change and improve outcomes for their students. The project team represents seven universities across all states and the Northern Territory. The institutions involved are very different in nature and include large city-based and rural and regional universities delivering a wide variety of teacher education courses. Course delivery is available to both full-time and part-time students, and also through distance or online modes. To meet the aims of the project, instruments were needed that would address what pre-service teachers at the end of their course knew and understood about mathematics teaching (Callingham et al, in press).

## PREVIOUS RESEARCH

Early attempts to measure teachers' knowledge were based on self-reports of mathematics courses taken. Such measures were found to have little or no relationship with students' outcomes (Mewborn, 2001). Ma (1999), in a cross-cultural study between China and the USA, described "Profound Understanding of Fundamental Mathematics (PUFM)" (p. 22) that she claimed was needed by elementary teachers to teach mathematics successfully. The identification of PUFM was based on interview data, and included detailed aspects of algorithms and processes common in elementary mathematics classes, such as long multiplication. In contrast, Even and Tirosh (2002) emphasised the importance of teachers' knowledge of their students' mathematical learning, also based on interview and observation approaches.

The largest body of reported work is that from Hill, Schilling and Ball (2004) who reported on initial attempts to develop measures of elementary teachers' mathematical knowledge for teaching. Using a combination of factor analyses and Item Response Theory (IRT) they developed three related constructs which addressed what they termed "Common Knowledge of Content" (CKC) which was mainly concerned with numbers and operations, "Knowledge of Students and Content" (KSC) and "Knowledge of Content" (KC) in patterns, functions and algebra. Of particular interest was that the "point of maximum information" was higher for the KSC scale than for the CKC or KC scales and had a greater mismatch with the ability measures of the sample of teachers tested. Using a refined version of their instrument, Hill, Rowan and Ball (2005) described Mathematical Knowledge for Teaching which they described as that "... mathematical knowledge used to carry out the *work of teaching mathematics*" (p. 373, italics in the original) including providing examples, explaining concepts, correcting work and using a range of representations of mathematical ideas. They also demonstrated some relationship between teachers' mathematical knowledge for teaching and students' outcomes, which has been the underlying assumption of all of this work but has proved remarkably difficult to establish.

Other teams have used somewhat different approaches. Watson (2001) used a profiling instrument intended to address all aspects of Shulman's (1987) knowledge types in the context of statistics and probability. Using this approach as a basis, Callingham and Watson (2011) described the development of measures of middle school teachers' pedagogical content knowledge in statistics. The items used were of two types. Teachers were asked to anticipate students' responses to questions, and to suggest interventions based on these anticipated responses, and also to authentic students' answers to survey questions. The instrument was a survey with open-ended responses, which were coded according to the complexity of the response. Findings suggested that teachers, in general, were better at identifying how students would most likely respond than they were at suggesting interventions.

Others have argued that the application of teachers' specialised mathematical knowledge for teaching is likely also to be the result of teachers' beliefs about teaching and the nature of the knowledge domain. Beswick (2011), for example, has argued from a theoretical perspective that if beliefs are regarded as anything that is "true", then beliefs and knowledge are identical. Beswick, Callingham and Watson (2011) built on this idea by using Rasch (1960) modelling to identify an underlying "thick" construct of teachers' knowledge that combined several of Shulman's knowledge types. The instrument used was a modified version of Watson's (2001) profile and comprised a combination of Likert scale items and open responses.

For the CEMENT project an instrument was required that could be administered online to a diversity of students across Australia. For this reason, the nature of the item types was constrained, and mainly consisted of multiple choice. Following on from Beswick, Callingham and Watson's

(2011) work, three scales were conceptualised: beliefs, mathematics content knowledge and mathematics pedagogical content knowledge. The remainder of this paper describes the development and initial results from the use of this instrument, the Test of Mathematics Teaching Knowledge, with pre-service teachers undertaking studies in primary education.

## METHOD

### *Item development*

Items were developed collaboratively by the project team at two meetings.

Beliefs items were taken from various sources to address identified aspects of teachers' beliefs about mathematics and its teaching. They included nine statements requiring respondents to indicate the extent of their agreement on 5-point Likert-type scales drawn from those used in previous studies (e.g., Thompson, 1984; Van Zoest, Jones, & Thornton, 1994). A single item asked participants to rate their confidence to teach mathematics at the grade levels for which they would be qualified to teach, also on a 5-point scale. The final scale was termed BELF.

Content knowledge items addressed the three content strands of the *Australian Curriculum – Mathematics* (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2011): Number and Algebra, Geometry and Measurement, and Statistics and Probability. Several of the participating universities were using some form of diagnostic test with their incoming students, and many of these items were adapted for the CEMENT project. Others were created to address perceived gaps. For pre-service primary teachers the expected standard was about Year 10 mathematics. The anticipated scale was named MCK (mathematics content knowledge).

The final set of items addressed what was termed pedagogical content knowledge (PCK). These items created much discussion, and were the most difficult to write. Items were considered from other places, including publicly released items from Hill et al (2004) but many of these did not address what the group considered to be important for PCK. The framework developed for the CEMENT project addressed four aspects of PCK deemed important based on the available literature, and prior experience of group members. The four aspects were:

1. Identifying students' errors or misconceptions;
2. Constructing or using tasks and tools for developing students' understanding;
3. Knowledge of a range of representations of a particular mathematical idea; and
4. Explaining ideas to students.

The first of these aspects aimed to tap into knowledge of students' thinking in mathematics, and was considered fundamental to good mathematics teaching. The second aspect targeted the notion of "affordances" – making productive use of stimulus material or artefacts to tap into mathematical ideas. Having a range of models and representations of mathematics is important not only because

it provides a variety of avenues for students to develop understanding, but also because this is an essential mathematical idea. Finally, the idea of explaining or communicating mathematically with students was considered the foundation of teaching. Items were developed around these PCK aspects across a variety of mathematical content areas.

No limit was placed on the number of items developed because it was recognised that a large pool of items would be needed. An example of the items from each of the Mathematics Teaching Knowledge domains is shown in Table 2.

Table 2

*Representative items from the Test of Mathematics Teaching Knowledge for Pre-service Teachers undertaking Studies in Primary Education*

<i>Item category</i>	<i>Example</i>
Beliefs (BELF)	Acknowledging multiple ways of mathematical thinking may confuse children.
Beliefs (BELF)	Students learn by practicing methods and procedures for performing mathematical tasks
Content knowledge (MCK)	Which one of the following contains a set of three fractions that are evenly spaced on a number line? <b>A)</b> $\frac{3}{6}, \frac{3}{5}, \frac{3}{4}$ <b>B)</b> $\frac{3}{4}, \frac{19}{24}, \frac{5}{6}$ <b>C)</b> $\frac{3}{4}, \frac{19}{24}, \frac{7}{8}$ <b>D)</b> $\frac{4}{5}, \frac{5}{6}, \frac{7}{8}$
Pedagogical Content Knowledge (PCK)	A Year 5 teacher asked her pupils to determine the value of the following calculation on their calculators: <div style="text-align: center;"> <span style="border: 1px solid black; padding: 2px;">2</span> <span style="border: 1px solid black; padding: 2px;">+</span> <span style="border: 1px solid black; padding: 2px;">3</span> <span style="border: 1px solid black; padding: 2px;">x</span> <span style="border: 1px solid black; padding: 2px;">4</span> <span style="border: 1px solid black; padding: 2px;">=</span> </div> <p>The class was surprised to find that some student calculators gave a result of 14, while others gave a result of 20. Which of the following best matches your likely response to this situation?</p> <ol style="list-style-type: none"> <li>Use the difference as a motivation to teach the students how to use correct order of operations, highlighting an acronym such as BODMAS.</li> <li>Show the students how to use parentheses or brackets when entering expressions into their calculators.</li> <li>Check school booklists and supplies to make sure that only one kind of calculator was available to students in the class.</li> <li>Ask the students to explain the different results, and use their explanations to discuss the order of operations as an arbitrary convention.</li> </ol>

All items were loaded into Qualtrics survey software ([www.qualtrics.com](http://www.qualtrics.com)) which provides a wide range of item types and can automatically score forced-choice type items. The Beliefs items were presented intact (10 items). Both MCK and PCK items were presented as timed sections of the test (20 minutes for each section) with a random selection of 10 items, five set as core items and five as a random selection from the remaining items in each section. Having 5 items from each of the MCK and PCK scales included in every test provided a means of equating each students' test responses

and also provided possible anchor items for future test delivery. These items were chosen after conducting a small-scale pilot study (Beswick & Callingham, in press; Callingham et al, 2011). The common items also included 3 items from a survey intended for pre-service secondary teachers to allow the opportunity to develop of a single scale of pre-service teachers' mathematics knowledge through common-item equating.

The Beliefs items were scored on a Likert scale from 1 to 5, where 5 represented the strongest agreement. No items were reversed. Item scoring for the MCK and PCK items was mostly dichotomous (right/wrong) but some of the PCK items were scored using a partial credit approach. For example, the PCK item shown above was scored as A = 1, B = 2, C = 0, D = 2 on the grounds that the situation provided several possible affordances to develop students' understanding and which route the teacher chose could depend on the class, the aim of the lesson or a variety of other practical considerations. The consequences of these decisions will be discussed further in the following sections.

#### *Other information*

In addition to the Test of Mathematics Teaching Knowledge, respondents were asked a variety of demographic information to provide background knowledge about the pre-service teachers for whom the test was intended. This information included home institution and course; date of intended graduation (as a de facto measure of how close they were to the end of the course); mode of study (on-campus, off-campus or some blend of the two); whether they were studying full-time or part-time; and some personal information including previous educational and mathematics background; and indigenous or non-English speaking background status.

#### *Sample*

The link to the online survey was sent to their own students by university representatives on the project team. Data were collected into a single data set. The intended group for the project was final-year pre-service teachers but several universities also included other groups to obtain information across their courses. The initial analyses were undertaken with a sample of 241 pre-service primary teachers from seven universities who responded to the test. Only 221 of these respondents (92%) answered the items – several logged on, completed the demographics and then decided not to go further. Of those who did attempt the remainder of the questions, it was noticeable that many answered the beliefs section but then did not attempt the mathematical items. Of the pre-service teachers who responded, 75 per cent (165/219) were studying full time. Surprisingly, however, 45 per cent (106/220) were undertaking their study at distance, with a further 40 per cent on campus. This result reflects the changing nature of tertiary study. Most were towards the end of their study period with over half (115/218; 53%) aiming to finish their degree in 2011 or 2012. Respondents had mainly undertaken a pre-tertiary mathematics course in Year 11 or

12. Less than 10 per cent were from Aboriginal or Torres Strait Islander backgrounds, or spoke another language rather than English at home.

*Analysis*

Rasch analysis was performed using Winsteps 3.69.1 software (Linacre, 2009), using the partial credit model. Four separate analyses were undertaken. First all items were scaled together to identify a general scale of pre-service teachers Mathematics Teaching Knowledge (MTK). Then three sub-scales were analysed individually to give a Beliefs scale (BELF), Mathematics Content Knowledge scale (MCK), and Pedagogical Content Knowledge scale (PCK). Scales were examined for overall fit to the model. Person ability measures were obtained and used in a variety of comparisons across sub-groups defined by the different demographic information.

**RESULTS**

Results are reported for each of the four scales, and then for a variety of comparison tests.

*MTK scale*

The overall scale consisted of 84 items, comprising 10 beliefs items, 45 MCT (content) items and 29 PCK items. Of these items, two MCT items were dropped from the analysis because all those students who had responded to these had done so correctly. The “bubble chart” (Bond & Fox, 2007) produced by the software to show the fit and difficulty levels of all items is shown in Figure 1. Only one item showed serious underfit. This was a beliefs item: “Acknowledging multiple ways of mathematical thinking may confuse children.” Responses to this item appeared to be inconsistent with all of the others; in other words, responses to this item were idiosyncratic or random.

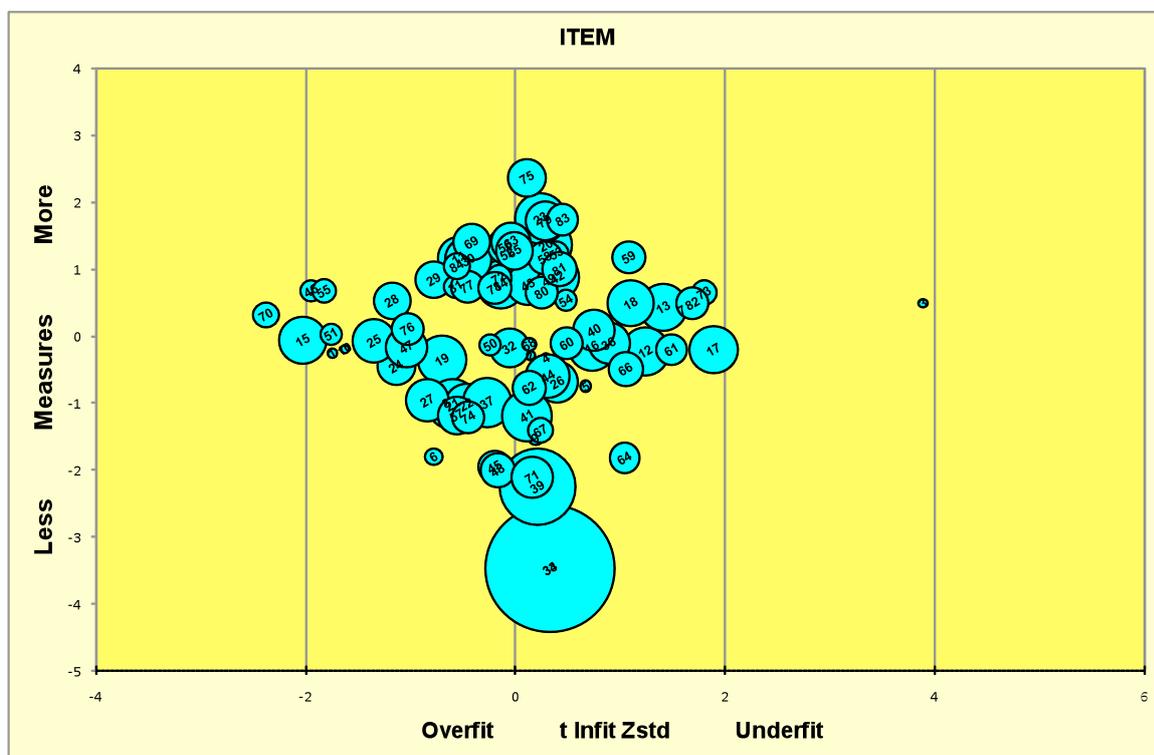


Figure 1. Bubble chart showing fit from all items on the overall MTK scale.

Summary fit statistics for this scale for persons and items are shown in Table 3. The Cronbach alpha reliability produced by Winsteps was high at 0.90. The overall fit to the Rasch model was good for both items and persons suggesting that the instrument provided satisfactory measures of pre-service primary teachers Mathematics Teaching Knowledge (MTK).

Table 3

*Summary statistics for items and persons for the MTK scale*

	N	Mean (logits)	Infit	z Infit	Outfit	z Outfit
Items	82	0.00	1.00	0.00	1.00	0.00
Persons	221	0.22	1.00	0.00	0.98	-0.1

*Subscales (BELF, MCK, PCK)*

The only misfitting item among the 10 beliefs items was the same one that misfitted in the full MTK scale. All other items appeared to work together to measure an underlying construct consistently.

Of the 43 MCK measured items, only two showed any degree of misfit and in both instances this was very small. Using the suggested “rule of thumb” limits of infit and outfit lying between  $\pm 1.3$  (Wright & Masters, 1982) these items would fit the model.

Among the 29 PCK items, only one showed any misfit and again that was small and would meet the criteria suggested by Wright and Masters (1982). This was an item about useful resources to develop young children’s subitising (recognition without counting of small numbers in a collection).

Summary statistics for all three subscales are shown in Table 4. The Cronbach alpha reliabilities were low for the MCK and PCK scales, probably because of the amount of missing data. Overall, it appeared that the instruments could provide reasonable measures that universities could use for monitoring purposes.

Table 4

*Summary statistics for items and persons for the three subscales*

Scale	N <sub>i</sub>	N <sub>p</sub>	Mean <sub>i</sub>	Mean <sub>p</sub>	Infit <sub>i</sub>	z Infit <sub>p</sub>	Outfit <sub>i</sub>	z Outfit <sub>i</sub>	Infit <sub>p</sub>	z Infit <sub>p</sub>	Outfit <sub>p</sub>	z Outfit <sub>p</sub>	Cr α
BELF	10	221	0.00	0.77	1.00	0.00	1.00	0.00	1.01	0.00	1.00	0.00	0.86
MCK	43	156	0.00	0.32	1.01	0.1	0.99	0.1	0.96	-0.1	0.93	0.00	0.64
PCK	29	115	0.00	-0.18	1.00	0.00	1.00	0.1	0.97	-0.1	0.98	0.00	0.65

From a Rasch measurement perspective, however, the scales did not separate respondents well along the variable. The person separation reliability statistics for all scales lay between 0.48 (MCK) and 0.36 (BELF). These figures suggest that the scales probably provide redundant data over a relatively narrow range of ability measure. One reason for this may be that the scales did not well target the intended group. As can be seen from the mean ability measures for each scale respondents found it easier to endorse the BELF items than to score well on the other two scales.

The pattern of decreasing mean abilities from BELF to MCK to PCK has been reported previously (Beswick, Callingham & Watson, 2011). This issue is addressed further in the discussion.

*Using the measures*

Because the aim of the project is to provide a tool that universities can use for monitoring purposes, a series of analyses was undertaken to compare groups, using the demographic variables. Because of the very low numbers of respondents from indigenous and non-English speaking backgrounds, these variables were omitted from further analyses.

Of particular interest was the educational and mathematics levels, and whether improved background knowledge impacted on outcomes. One way ANOVA analyses using these variables showed small differences only in the MCK scale. For EDLEVEL the MCK mean difference was significant at the 0.05 level ( $F = 2.56$ ,  $df = 5$ ,  $p = 0.03$ ) and for MATHLEVEL it was significant at the 0.01 level ( $F = 3.381$ ,  $df = 6$ ,  $p = 0.004$ ). No other variable, including mode of study, full-time or part-time, or year of completion showed any statistically significant difference.

There was some indication that means on the overall MTK scale did improve slightly the closer pre-service teachers got to graduation. The subscales showed different patterns. On MCK (content knowledge) there appeared to be a sharp improvement early in the tertiary experience that plateaued across the years of the course. The BELF scale, in contrast, varied little across the years towards graduation but the trend appeared to be slightly downwards. The PCK scale showed the greatest variation, and this might be due to recency of experience with a mathematics education unit within the degree. None of these findings, however, showed any statistical significance. The graph in Figure 2 provides a summary of these findings.

The graph also shows the mean differences between the scales. The BELF scale was always higher than either MCK or PCK. Content knowledge (MCK) and pedagogical content knowledge (PCK) appeared similar early in the degree (4 years from graduation) but content knowledge improved quickly whereas PCK was more variable and did not appear to improve greatly across the period of training. There are two possibilities to explain this finding. First, the one year from graduation included students undertaking a one-year Diploma of Education course. It is possible that this skewed the results, especially since the PCK level was similar to that of pre-service teachers closer to the start of their degree. The second possibility is related to the placement of mathematics education units within the degree, as indicated previously. These conjectures will be considered through further data analysis.

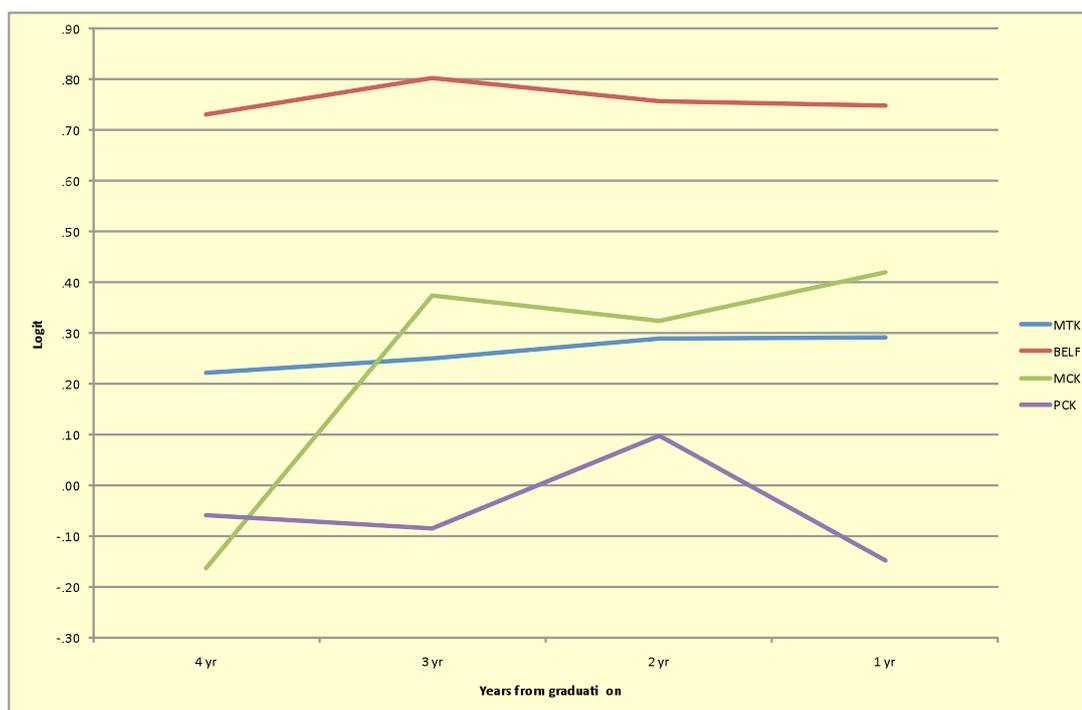


Figure 2. Subscales by years from graduation.

## DISCUSSION

The purpose of this project is to develop an instrument that teacher education faculties could use to monitor their mathematics education programs. A long-term aim is to provide benchmarks for institutions that indicate the kinds of mathematics teaching knowledge that graduating students will need to meet accreditation standards. To this end the Test of Mathematics Teaching Knowledge was developed to address three key domains: beliefs about mathematics and its teaching; mathematical content knowledge; and pedagogical content knowledge for teaching mathematics. Initial results are promising. Apart from one item, “Acknowledging multiple ways of mathematical thinking may confuse children,” all items appeared to work consistently to provide information about a thick construct of Mathematics Teaching Knowledge. Three subscales also fitted the Rasch model well, indicating that they could be used independently to investigate specific facets of a mathematics education course. The scale is still far from ideal, however. More “easy” PCK items would enhance the scale. Scoring items using a form of partial credit, as suggested for the PCK item in Table 2 did help to reduce the difficulty level of the items, with the partial-credit scored items among the lower difficulty level items on the PCK scale. Conversely, it appears difficult to develop Likert scale items about beliefs that are difficult for pre-service teachers to endorse. The data shown in Figure 2 suggests that beliefs are relatively fixed and are not easily influenced by teacher education courses. The challenge for mathematics teacher educators appears to be to change beliefs into effective teacher action. Mathematics content knowledge appears susceptible to

change, as shown in Figure 2, but PCK is more difficult to develop. These findings are consistent with those of Hill et al (2004).

It is intended to ask primary teachers and mathematics educators themselves to attempt the Test of Mathematics Teaching Knowledge. It is expected that these groups would perform better on the PCK scale in particular, and with a better match to the group of test-takers the Person Separation Reliability might improve. It is hoped that this expert group will provide aspirational benchmarks for teacher education programs.

There are also further analyses to undertake in relation to the nature and structure of the pre-service teacher education programs, and the relevant performances of pre-service teachers on the different scales. In particular, there is interest in establishing whether short (1 to 2 years) courses for primary pre-service teacher education provide the same standards of Mathematics Teaching Knowledge as the more traditional 4-year Bachelor of Education.

Finally, there is also interest in using the items diagnostically in order to improve programs in mathematics education. Initial consideration does seem to indicate that students from different institutions respond in dissimilar ways on some items, most likely reflecting different emphases during their course.

These initial results appear to indicate that the goals of the project are not unrealistic and that the instrument in its current form is usable and can potentially provide useful information.

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