Robust State Estimation in Distribution Networks

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Abstract— In this paper a new approach to state estimation in distribution networks is proposed. This approach is more robust against large uncertainties of the state estimation inputs than the conventional method. Traditionally, the goal of state estimation was to estimate the exact value of network parameters, such as voltages and currents. This works well in transmission networks where many real time measurements are available. In distribution networks, however, only few real-time measurements are available. This means that the estimated state can be significantly different from the actual network state. Therefore, the focus of the proposed robust state estimation is shifted from estimating the exact values of the network parameters to the confidence that these parameters are within their respective constraints. This approach is able to provide useful results for distribution network operation, even if large uncertainties are present in the estimated network state.

Index Terms— Distribution network state estimation, load uncertainty, uncertainty quantification

I. Introduction

In distribution networks electric power is traditionally delivered from the substation to the consumer. However, with the increasing amount of distributed generation from sources such as solar and wind, bidirectional power flows are possible. This can result in significant voltage and security concerns [1]. The constraints of a network are defined by its physical properties (e.g. thermal limits of network components) and regulations (e.g. the voltage compliance range) [2]. However, the distribution network operator cannot take any control actions (such as adjusting transformer taps, switching on or off capacitor banks, etc. [3-5]) without accurate information about the network state. The state of a power network is uniquely identified by the voltage magnitude and angle at every bus [6]. All other network parameters such as currents and power-flows can be calculated from the network state. The network state is estimated by the state estimation (SE) process from the available set of measurements [7, 8]. SE is a standard procedure in transmission networks where numerous real-time measurements are available. In distribution networks, however, only a limited number of real-time measurements are usually available. In order to make the SE possible pseudo-measurements are often used in the absence of real-time measurements. A pseudo-measurement represents the load and generation connected to particular bus in a network and is derived from historical data. Since it is not possible to accurately predict the load and generation at a particular bus from historical data, pseudo-measurements usually have large margins of error associated with them. As a result, the estimated network state can contain a significant amount of uncertainty if a large number of pseudo-measurements are used in the SE [9, 10]. The aim of current SE methods is to estimate the exact value of the network parameters [11-13]. However, due to the generally large amount of uncertainty in the distribution network SE, the estimated state can be significantly different from the actual network state[14]. This makes it difficult to apply the conventional SE approach to distribution networks. Hence, a new approach to SE is required that is robust against large uncertainties in the distribution network SE.

The method proposed in this paper takes a different approach to the SE problem. The main concern of a distribution network operator is to keep the network within its constraints. This means that it is not important for the distribution network operator to know the exact value of the network parameters, but rather the confidence that these parameters are within their respective constraints. Hence, in the proposed approach, the focus is shifted from estimating the exact value of the network parameters, to estimating the confidence that these parameters are within their respective constraints. This approach can provide practical information for the distribution network operation, and is robust against large SE uncertainties. The confidence values of the estimated parameters are calculated from their probability density functions (PDFs) and their respective constraints. The practicality of the proposed approach is demonstrated by a case study performed on the IEEE 34-bus test feeder [15].

The rest of the paper is organized as follows. Section II presents the proposed robust SE method. In Section III, a case study and simulation results are provided. A discussion on the application of the proposed approach to larger networks is given in Section IV, and Section V concludes the paper.

II. Robust State Estimation

In this section the proposed robust SE is explained.

A. Conventional State Estimation

The SE process uses the available set of measurements to calculate the network state. If a network has a number of n busses its state is defined by $2n$ state variables (voltage magnitudes and angles at every bus). However, the angle at one particular bus is commonly set to zero ($\theta_1 = 0$) since it is used as a reference. Hence, the state of a network is uniquely identified by the state vector

$$\boldsymbol{x}^T = [\theta_2, \ldots, \theta_n, V_1, \ldots, V_n], \tag{1}$$

where, θ is the voltage angle and V is the voltage magnitude. The relationship between the measurement vector, $\boldsymbol{z}$, (which consists of all measurements used in the SE) and the network state vector, $\boldsymbol{x}$, can be expressed by

$$\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{e}, \tag{2}$$

where $\boldsymbol{e}$ is the vector of measurement errors and $\boldsymbol{h}(\boldsymbol{x})$ is a vector of nonlinear functions that relates the state vector, $\boldsymbol{x}$, to the measurement vector, $\boldsymbol{z}$.

The weighted least squares method (WLS) is the most popular approach to estimate the network state and its objective is to minimize the following function [6]:

$$\min \boldsymbol{J}(\boldsymbol{x}) = [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})]^{\mathrm{T}} \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})], \tag{3}$$

$$\boldsymbol{R} = diag[\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2], \tag{4}$$

where σ^2 is the measurement variance; $\boldsymbol{R}$ is the measurement error covariance matrix; and the number of measurements which are used in the SE is given by m. The WLS method is an iterative process given by

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k + \boldsymbol{G}(\boldsymbol{x}^k)^{-1} \boldsymbol{H}^T(\boldsymbol{x}^k) \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] \tag{5}$$

$$\boldsymbol{H}(\boldsymbol{x}) = \left[\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right] \tag{6}$$

$$\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{H}^T(\boldsymbol{x}) \boldsymbol{R}^{-1} \boldsymbol{H}(\boldsymbol{x}) \tag{7}$$

where $\boldsymbol{H}$ and $\boldsymbol{G}$ are the Jacobian and Gain matrices, respectively.

B. Robust State Estimation

In the proposed robust SE approach, the PDFs of the estimated parameters have to be obtained first. This can be done by a variety of methods that have been proposed in literature [16-19]. Which method is most suitable for a particular network, has to be decided on a case by case basis. It is, however, important to note that the proposed approach does not depend on the method chosen to calculate the PDFs.

In this paper, a method based on the gain matrix is used to calculate the PDF of the estimated parameters under the assumption of Gaussian distributed inputs [20]. In this method, the variance of the estimated state variables is calculated by

$$\boldsymbol{cov}(\boldsymbol{x}) = [\boldsymbol{H}^T(\boldsymbol{x}) \boldsymbol{R}^{-1} \boldsymbol{H}(\boldsymbol{x})]^{-1}, \tag{8}$$

where the variance of the estimated state variables is represented by the diagonal elements of $\boldsymbol{cov}(\boldsymbol{x})$ [20]. The matrix $\boldsymbol{cov}(\boldsymbol{x})$ can also be used to calculate the variance of all other estimated parameters [21]. Once the variance of an estimated parameter is known, its PDF is given by

$$PDF_i(a_i) = \frac{1}{\sqrt{2\pi\sigma_i{}^2}}\, e^{-\frac{(a_i - E_i)^2}{2\sigma_i{}^2}}, \tag{9}$$

where σ_i^2 is the variance of i for $i = [1,2, \ldots, i, \ldots, N]$; N is the total number of estimated network parameters; E_i is the estimated value of the network parameter i; and a_i is a possible value of the parameter i.

C. Calculating the Confidence

After calculating the PDF of all estimated parameters, the confidence that these parameters are within their respective constraints are calculated. The constraints of the i^{th} estimated parameter are given by $a_{i,min}$ and $a_{i,max}$, representing the minimum and maximum values of the i^{th} estimated parameter respectively.

The confidence value of the i^{th} estimated parameter is calculated by taking the integral of its PDF_i between its constraints $a_{i,min}$ and $a_{i,max}$. This if formally defined by

$$Conf_i = \int_{a_{i,low}}^{a_{i,high}} PDF_i(a_i)\, da_i \tag{10}$$

where a_i is a possible value of the i^{th} estimated parameter, PDF_i is the probabilitiy density function of the i^{th} estimated parameter, and $Conf_i$ represents the confidence that the true value of the i^{th} estimated parameter lies within its constraints. The higher the value of $Conf_i$, the lower the risk that the true value is outside its constraints, and vice versa.

III. Case Studies

A. Test feeder

The following case study shows how the proposed approach can be applied to a distribution network and is performed on the IEEE 34 bus-test feeder.

For simplicity, it is assumed that connected loads and generators are three-phase and balanced. However, it is important to understand, that the proposed method can also be applied to an unbalanced network in the same way as it is applied to a balanced one, if three-phase measurements and sufficient information about phase connections of loads and distributed generators is available. The voltage compliance range is set to $\pm 5\%$ of the nominal network voltage; measurement errors are assumed to be normally distributed; pseudo-measurements with the accuracy of $\pm 50\%$ are used to represent all loads; pseudo-measurements with the accuracy of $\pm 100\%$ are used to represent distributed generation (such as wind and solar); and the voltage at Bus 1 is measured with the accuracy of ±1%. The standard deviation of a measurement is calculated as the product of one-third of the measurement accuracy and the expected value [22].

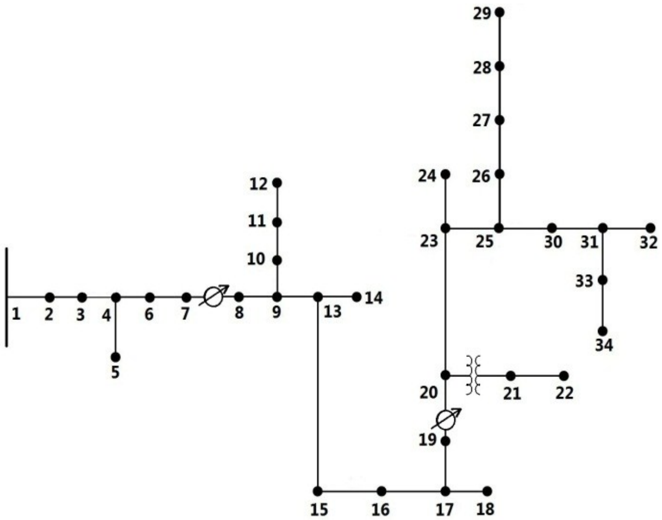

Fig. 1. The 34-bus test feeder.

For this case study we assume that the voltage regulating devices are operating normally and are able to maintain a voltage at the regulated busses within the specified bandwidth. Hence, the voltage at the regulated busses 8 and 20 is assumed to be kept at the set point. In order to take variations around the set point, due to inaccurate regulation, into account, the regulators are modeled as additional measurements with the accuracy of ±1%. This accuracy value represents the bandwidth of the voltage regulators specified in [32].

However, the actual positions of the voltage regulator taps are not known. For this reason, the additional measurement points used to model the voltage regulators, cannot provide any information about voltages at busses that are hierarchically higher than the voltage controlled bus (Bus 1 is assumed to be at the top of the hierarchy). Hence, the network is separated into sub-networks at the voltage controlled busses in order to improve the accuracy of the SE. The PDF of the estimated parameters are calculated for each sub-network starting with the hierarchically lowest sub-network. The resulting power flow at the voltage controlled bus that connects to the next sub-network and its PDF are used as inputs for the next network section.

B. Test Cases

In this study, two different cases are simulated. Case 1 considers the 34-bus test feeder under the loading condition specified in [15] without distributed generation. In Case 2 the network loading is reduced to 30% of the specified loads and distributed generation equal to 20% of the specified loads is added. These values are chosen for demonstration purposes only. For a real case these values should be known to the network operator.

Fig. 2 and Fig. 3 show the estimated voltage profiles of Case 1 and Case 2, respectively. The increase in voltage that can be seen in Fig. 3 can be attributed to a combination of the distributed generation and reactive power injected by two shunt capacitors located at busses 27 and 29.

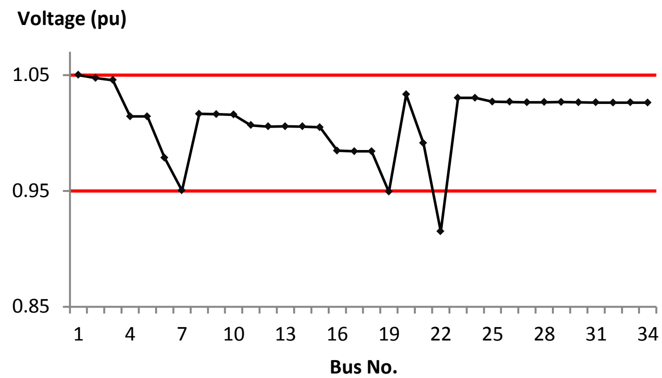

Fig. 2 The voltage profile for the 34-bus test feeder under base loading condition without distributed generation (Case 1).

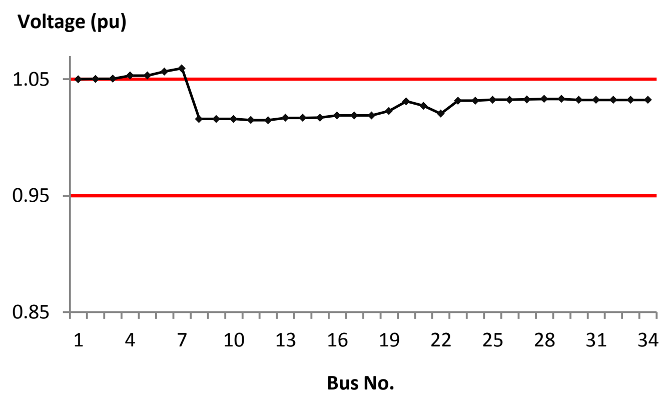

Fig. 3. The voltage profile for the 34-bus test feeder under low loading condition with distributed generation (Case 2).

Considering that only the voltage at Bus 1 is actually measured, it is clear that the estimated network state will contain a significant amount of uncertainty. However, no information about the accuracy of the estimate network state is provided by the results of the conventional SE shown in Fig. 2 and Fig. 3. This highlights that it is difficult to make decisions on the network operation, based on the conventional SE result, without any additional information about the SE accuracy.

Next, the proposed robust SE approach is applied to the test cases. An indication of the uncertainty in the SE results is provided in Figs 4 and 5 by the bars that represent the 95% confidence interval of the estimated voltage magnitudes. The calculated confidence values for the estimated bus voltages are shown in Table I and Table II. Busses with a confidence value lower than 100% are highlighted in gray.

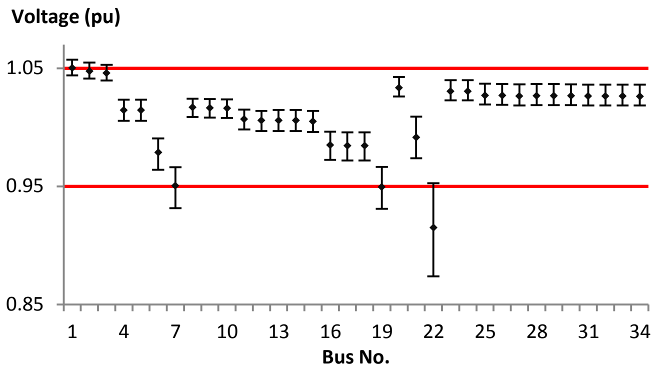

Fig. 4. The 95% confidence interval for the base loading condition without distributed generation (Case 1).

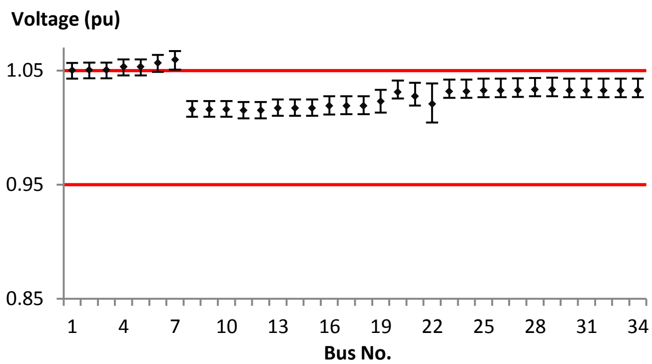

Fig. 5. The 95% confidence interval for the low loading condition with distributed generation (Case 2).

Table I: Confidence values for Case 1

Bus	Conf.	Bus	Conf.	Bus	Conf.
1	49%	13	100%	25	100%
2	76%	14	100%	26	100%
3	90%	15	100%	27	100%
4	100%	16	100%	28	100%
5	100%	17	100%	29	100%
6	100%	18	100%	30	100%
7	51%	19	45%	31	100%
8	100%	20	100%	32	100%
9	100%	21	100%	33	100%
10	100%	22	4%	34	100%
11	100%	23	100%		
12	100%	24	100%		

Table II: Confidence values for Case 2

Bus	Conf.	Bus	Conf.	Bus	Conf.
1	51%	13	100%	25	100%
2	49%	14	100%	26	100%
3	47%	15	100%	27	100%
4	20%	16	100%	28	100%
5	20%	17	100%	29	100%
6	4%	18	100%	30	100%
7	2%	19	100%	31	100%
8	100%	20	100%	32	100%
9	100%	21	100%	33	100%
10	100%	22	100%	34	100%
11	100%	23	100%		
12	100%	24	100%		

The confidence that a particular parameter is within its constraints can be low due to either the proximity of the parameter estimate to its constraints, or a low accuracy of the estimate. From Fig. 2 and Fig. 3 it is clear that only the proximity of the parameter estimates to their constraints is provided by the results of the conventional SE approach. The proposed robust SE on the other hand, takes both the proximity of the estimates to their constraints as well as the accuracy of the parameter estimates into account. As a result it is possible for the distribution network operator to make informed decisions on the network operation, without requiring a large number of real-time measurements.

The busses highlighted in gray (in Table I and Table II) are critical busses which may require control actions from the distribution network operator. A possible control action for Case 1 would be to increase the set point of the voltage regulator at Bus 7 to improve the confidence at Bus 19. For Case 2 a possible control action would be to decrease the voltage at the substation (Bus 1) in order to improve the confidence values at busses 1-7.

IV. Discussion

A. Application to larger networks

The execution time of the proposed approach largely depends on the method chosen to calculate the PDFs of the estimated parameters. For instance, the method used in this paper is based on the gain matrix which is already calculated during the WLS method. Hence, if this method is used to perform the conventional SE, nearly no additional computation time is required, to calculate the PDFs of the estimated parameters.

However, if the proposed robust SE method is applied to larger networks it can be difficult to interpret the results due to the large number of estimated parameters. A potential solution to this problem is to visualize the results graphically. To illustrate this approach, the results for Case 1 and Case 2 are provided as heat maps in Figs 6 and 7.

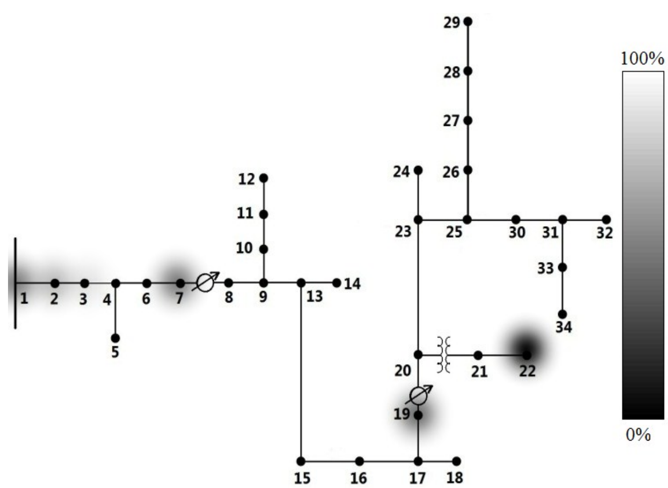

Fig. 6. Graphical representation of the results for Case 1

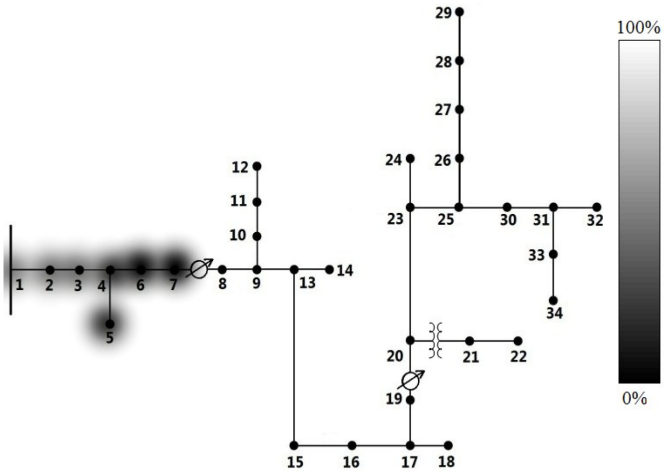

Fig. 7. Graphical representation of the results for Case 2

The dark areas in Figs 6 and 7 represent busses with low confidence values. This graphical representation of the robust SE results makes it possible to immediately identify critical areas in a network.

V. Conclusion

A new approach to state estimation has been presented in this paper. This approach is more robust against the large amount of uncertainty which is generally apparent in distribution network state estimation inputs. It was shown that the conventional state estimation approach cannot provide any information about the uncertainty in the state estimation result. This makes it difficult for the conventional approach to be applied to distribution networks. The proposed robust state estimation method, on the other hand, focuses on providing the confidence that the estimated parameters are within their respective constraints. This approach takes not only the proximity of the estimated values to their constraints into account, but also the accuracy of the estimates. A case study was performed on the IEEE 34-bus test feeder to show how this approach can be applied to a distribution network. The results of the case study demonstrate that the proposed approach is able to provide useful information about the network state even if a large amount of uncertainty is apparent in the SE inputs. A discussion on the application of the proposed method to larger networks has been given along with an example for a graphical representation of the robust state estimation results. This graphical representation makes it possible to quickly identify critical areas in the network. Future work will focus on the application of the proposed method to actual distribution networks as well as networks with a large number of busses.

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REFERENCES

[1] M. Powalko, A. G. Orths, H. Abildgaard, P. B. Eriksen, K. Rudion, I. I. Golub*, et al.*, "System observability indices for optimal placement of PMU measurements," in *IEEE Power and Energy Society General Meeting*, San Diego, CA 2012, pp. 1-6.

[2] B. Brinkmann and M. Negnevitsky, "A Probabilistic Approach to Observability of Distribution Networks," *Accepted for publication in IEEE Trans. on Power Syst.,* pp. 1-10, June 2016.

[3] L. Ramesh, S. P. Chowdhury, S. Chowdhury, a. a. Natarajan, Y. H. Song, and P. K. Goswami, "Distributed state estimation technique for active distribution networks," in *42nd International Universities Power Engineering Conference*, 2007, pp. 861-866.

[4] C. M. Hird, H. Leite, N. Jenkins, and H. Li, "Network voltage controller for distributed generation," in *Generation, Transmission and Distribution*, March 2004 pp. 150-156.

[5] F. Jiyuan and S. Borlase, "The evolution of distribution," *IEEE, Power and Energy Magazine,* vol. 7, pp. 63-68, 2009.

[6] A. Abur and A. G. Expósito, *Power System State Estimation Theory and Implementation*. New York: Marcel Dekker, Inc., 2004.

[7] F. C. Schweppe and J. Wildes, "Power System Static-State Estimation, Part I: Exact Model," *IEEE Trans. Power App. Syst.,* vol. PAS-89, pp. 120-125, Jan. 1970.

[8] C. N. Lu, J. H. Teng, and W. H. E. Liu, "Distribution system state estimation," *IEEE Trans. on Power Syst.,* vol. 10, pp. 229-240, Feb 1995

[9] L. Ke, "State estimation for power distribution system and measurement impacts," *IEEE Trans. on Power Syst.,* vol. 11, pp. 911-916, May 1996

[10] B. Brinkmann, M. Negnevitsky, T. Yee, and T. Nguyen, "An Observability Index for Distribution Networks using Information Entropy," in *Australasian Universities Power Engineering Conf. (AUPEC)*, Wollongong, NSW, 2015, pp. 1-6.

[11] J. Wu, Y. He, and N. Jenkins, "A robust state estimator for medium voltage distribution networks," *IEEE Trans. on Power Syst.,* vol. 28, pp. 1008-1016, 2013.

[12] L. Liu and J. H. He, "Application of weighted least square algorithm in distribution network state estimation with finite measurement information," in *International Conference on Power System Technology (POWERCON)*, 2014, pp. 134-138.

[13] D. Ablakovic, I. Dzafic, R. A. Jabr, and B. C. Pal, "Experience in distribution state estimation preparation and operation in complex radial distribution networks," in *IEEE PES General Meeting*, 2014, pp. 1-5.

[14] B. Brinkmann and M. Negnevitsky, "A Probabilistic Approach to Observability of Distribution Networks," *IEEE Trans. on Power Syst.,* vol. PP, pp. 1-10, 2016.

[15] (05 August 2013). *IEEE PES Radial Distribution Test Feeders* [Online]. Available: http://ewh.ieee.org/soc/pes/dsacom/testfeeders/

[16] M. Strelec, P. Janecek, D. Georgiev, A. Zapotocka, and E. Janecek, "Backward/forward probabilistic network state estimation tool and its real world validation," in *56th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON)*, 2015, pp. 1-6.

[17] A. K. Ghosh, D. L. Lubkeman, M. J. Downey, and R. H. Jones, "Distribution circuit state estimation using a probabilistic approach," *IEEE Trans. on Power Syst.,* vol. 12, pp. 45-51, 1997.

[18] G. Valverde, A. T. Saric, and V. Terzija, "Stochastic Monitoring of Distribution Networks Including Correlated Input Variables," *IEEE Trans. on Power Syst.,* vol. 28, pp. 246-255, 2013.

[19] L. Guang, Z. Ning, T. Ferryman, and F. Tuffner, "Uncertainty quantification in state estimation using the probabilistic collocation method," in *IEEE/PES Power Systems Conference and Exposition (PSCE)*, 2011, pp. 1-8.

[20] G. M. Mathews, "The accuracy of factored nonlinear weighted least squares state estimation," in *IEEE Int. Energy Conf. and Exhibition (ENERGYCON)*, Florence, 2012, pp. 860-866.

[21] J. J. Grainger and W. D. Stevenson, *Power system analysis*: New York: McGraw-Hill, International ed. , 1994.

[22] R. Singh, B. C. Pal, and R. B. Vinter, "Measurement Placement in Distribution System State Estimation," *IEEE Trans. on Power Syst.,* vol. 24, pp. 668-675, May 2009.