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Students' perceptions of the concept of function: The case of Turkish students attending vocational high school on industry

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Abstract

The purpose of the study was to investigate students' perceptions of the concept of functions in vocational high schools on industry. The participants were tenth grade students (n = 11) studying in the department of Computer Technology and Electrics and Electronics. A 10-item diagnostic test was designed to assess students' understanding of the concept of function. After analyzing students' written works for the test, three students were selected to be interviewed to reveal more about their ways of thinking in deciding whether a representation is a function or not. Results have revealed that the students had very weak perceptions of the function concept.

Keywords: Vocational and technical secondary education; concept of function; students' perceptions.

1. The Concept of Function

The concept of function is a central theme in school mathematics and it is essential in all of mathematics, particularly to the algebra curriculum (Bednarz, Kieran, & Lee, 1996; Harel & Dubinsky, 1992; National Council of Teachers of Mathematics [NCTM], 1989, 2000). It is critical in mathematics education and plays a central role in mathematics since it serves as main role in a students' understanding of mathematics (Harel & Dubinsky, 1992). It has been defined in various ways by many (e.g., Leibniz, 1692; Bernoulli, 1718; Euler, 1748; Dirichlet, 1829; Bourbaki, 1939) throughout its history (Kleiner, 1989). Among them Bourbaki' definition which "the rule of correspondence" has remained dominant in mathematics and influenced the teaching and learning of functions at secondary school (Kwari, 2007). However, this definition which based on correspondence of two sets (the domain and the co domain) is not sufficient for students to conceptualize the function concept (Vinner, 1983).

The concept of function is represented many different ways such as; a formula, an expression, a verbal description, a rule, a set of correspondence and so on. It is difficult to predict just what of these forms the emphasis on functions will take in the future, but they may be moved from one form to another (Cooney & Wilson, 1993). Unfortunately, for some students this is not understandable. For instance, "y = 2x" may be a function as long as you do not graph it.

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However, students can compare and translate among representational illustrations of functions just if they have a strong process conception of function (Harel & Dubinsky, 1992). Then, "a major goal of the secondary and collegiate curriculum should be developed in students a *sense for function*" (Eisenburg, 1992, p. 154); because it seems clear that functions will continue to play a prominent role in school mathematics (Cooney & Wilson, 1993).

2. Vocational and Technical Secondary Education

Vocational high school on industry is a type of vocational and technical secondary education institutions in Turkey. Their aim is to "educate the pupils as manpower for business and professional branches in line with the objectives of general secondary education and prepare them for tertiary education" (Milli Eğitim Bakanlığı [MEB], 1991). Training in these schools varies among different branches such as computer technology, electrics and electronics, wood technology etc. In vocational and technical education institutions, the students receive only theoretical education including fundamental science and mathematics courses within the school in 9th grade. However, mediated by the curriculum in these schools, students in vocational high schools have performance levels below those in general secondary education institutions in terms of social studies, mathematics and science. Although mathematics is considered one of the most important subject areas for students in vocational high school on industry, they consider it as very difficult and think that it is almost impossible to go beyond arithmetic. Algebra is clearly a fundamental part of secondary mathematics and one of its central concepts is functions. One way or another, we need to find ways to make algebra, particularly its fundamental concepts like functions accessible to those students. Doing this, we first need to study to what extent students in these institutions conceive algebraic concepts. Such knowledge would have valuable insights for teaching. Therefore, the purpose of the present study was to investigate students' perceptions of the concept of functions in vocational high schools on industry. The study was guided by the following research question: "What perceptions of the function concept do the industrial vocational high school students have?"

3. Methodology

The participants of the study were 11 tenth grade students studying at department of Computer Technology (CT) and Electrics and Electronics (EE) in an industrial vocational high school (see Table 1). The students had been studying functions for over two years, since they were first introduced to the concept of function in the ninth grade. They had four mathematics lessons (40 min. each) per week and their academic backgrounds (mathematics exam results, mathematics interest, their participation in the classes etc.) were similar.

Table 1: Distribution of the participants according to the departments they were enrolled and their gender

	Male	Female	Total
CT	4	1	5
EE	5	1	6

The data in this study were obtained using a 10-item diagnostic test which was designed to assess students' understanding of the concept of function (see Figure 1 for sample items). Each item was intended to probe one representation of a function (i.e., set correspondence diagrams, set of ordered pairs, equations, or graphs). The students were given the test and asked whether the representations in the items show functions or not. They were also asked to give reasons for their answers. The test was administered by the first author and took about 30 minutes to answer for each student. Students' responses were analyzed and grouped as "Yes", "No" or "No idea." After this categorization, their answers were coded like "this is a function, because of 'x' and 'y'" or "this is a function because an expression is a function".

3. Results

The results are presented according to the representational type of function concepts (i.e., set correspondence diagrams, set of ordered pairs, equations, and graphs) by sample items.

3.1. Set correspondence diagrams

The representation in items 3 shown in Figure 1 is not a function, because there is an element in the domain not corresponding to any element in range and there is an element in the domain corresponding to two elements in the

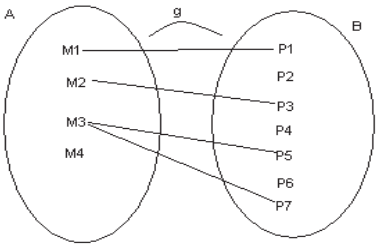
range. Almost all of the students (n = 9) were sure that this representation shows a function except for one students who didn't have an idea and one student whose answer was yes. Her explanation was that "I know if it is a function but I cannot exactly remember why". From the other students' responses, it was obvious that their way of thinking about functions was, in one way or another, mediated by the set-correspondence definition of the concept of function.

Decide whether the following relations are functions of not.

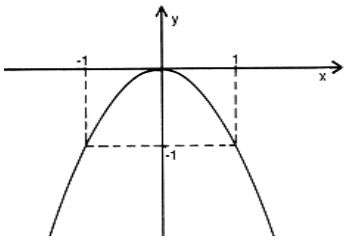
1) $A = \{1,2,3,4\}$ and $f: A \to R$, $f = \{(1,1),(2,1),(2,2),(4,3)\}$

2) $x = 3y - 4$

3)



4) $f: R \to R$



5) Angle of Sunlight strikes on Earth vs. Temperature (°C)

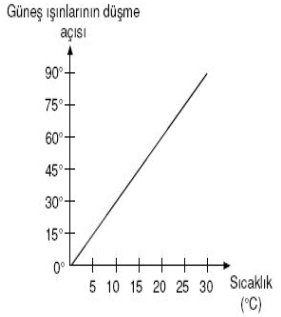
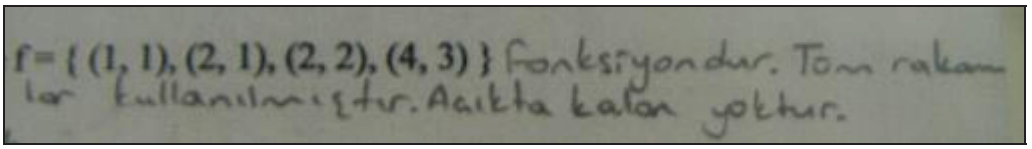


Figure 1: Sample items from the test

3.2. Set of ordered pairs

There were two items about set of ordered pair representations in the function test. One of them was the first item in Figure 1. While one student were not sure if it is a function of not, five thought that it is a function and the remaining five thought not. Students' reasons, who thought the representation was not a function, were quite similar to each other: "*Some elements of the two clusters did not match with each other*". However, perceptions of the other students, who think the representation was a function, were different from each other. Their reasons grouped under three categories: (1) "*it is a function because of "f" and "R"*, (2) "*it is familiar to me*", and (3) (as shown in Figure 2) "*all elements of the two clusters are matched with each other*".

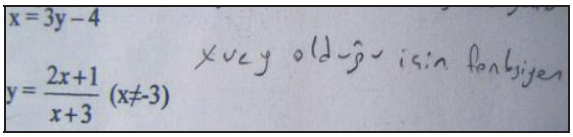


[… is a function. All of the numbers are used. None remained unused.]

Figure 2: A sample answer of a student to the set of ordered pair item

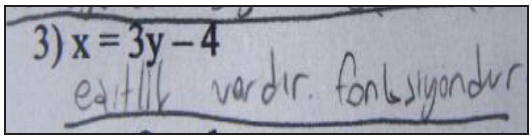
3.3. Equations

One of the two items showing symbolic (equation) representations of the function concept in the test was "$x = 3y - 4$". Most of the participants thought that the expression was a function while some could not decide (see Table 2). While the representation was a function due to "*x*" and "*y*" for four of them (see Figure 3a), it was because of the "equivalence (=)" for three (see Figure 3b).



[It is a function because there are x and y]

Figure 3a: A sample answer of a student to the equation item



[It is equality. So, It is a function.]

Figure 3b: A sample answer of a student to the equation item

Table 2: Distribution of students' answer whether the equation "x = 3y – 4" is a function

	Yes	No	N/I	N/R	Total
# of Students	7	2	1	1	11

N/I: No Idea; N/R: No Response

3.4. Graphs

In the test there were 5 items containing graphical representations of function concept. Two of them were the 4th and 5th items in the Figure 1. Although there were no students who directly answered that the graphical representation in the item 4 is a function, there were five students whom either stated that they had no idea (n = 3) or have no response at all (n = 2). There of the six students' who stated that the graphical representation was a function because they remembered the very same figure in their textbooks. On the other hand, one of the students though that it is a function because of the letter "*f*" in $f: R \to R$. Another student thought that it is a function because the graph moves in an even manner.

Another question containing graphical representations of function concept was the one in Figure 4. Most of the students had no idea about the representation shown in the item 5 (see Table 3).

Table 3: Distribution of students' answer whether the relation that the graph in item 5 shows a function

	Yes	No	N/I	N/R	Total
# of Students	4	1	5	1	11

N/I: No Idea; N/R: No Response

One of the students expressed that *"This is not a function, because I remember like that"*. The students' thought that it was a function had the same reason: "*It is a function because there is an equal rate of increase in the graph"* (see Figure 4).

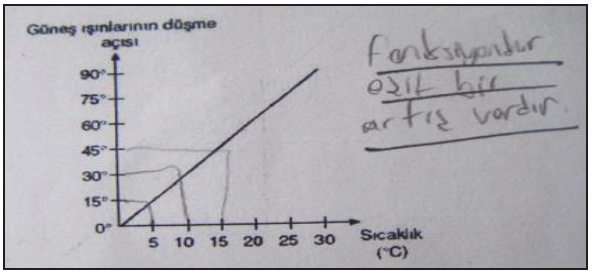


Figure 4: A sample answer of a student to the linear graph

4. Conclusion

In the present study, students attempted to focus on the set corresponded definition of the concept of function when deciding whether a given representation was a function or not; but they could not do it coherently. In parallel to Akkoç (2003), they were more successful in *set correspondence diagrams* and *set of ordered pairs* than in *equations* and *graphs*. However, *equations* and *graphical representations* were a dilemma for them since, they could not decide whether an *equation* and a *graph* was a function or not. Graphical representations were the students' biggest weakness, in the study. Generally, they tried to decide and thought carefully whether those were a function or not, but by irrelevant decisions.

In the Turkish curriculum, the concept of function is first introduced (and defined) in ninth grade mathematics textbook, mediated an activity (MEB, 2007; p.70). In this activity there is a problem situation as four friends go to the restaurant to have lunch. They eat a meal according to two conditions as: (1) All people must eat and (2) One person must eat only one meal. The problem situation also includes two sets, one consists of four friends: "soner, yasar, okan and hakan" and the other one consists of seven meals that in the restaurant: "shish kebab, dry bean, meatball, spinach, potato, staffed vegetable and celery." Then, a set correspondence is presented and it is asked to students: (a) Write the person and the meal he eats in set of ordered pairs. (b) Is there any who do not eat meal? (c) Is there any who eat more than one meal? (d) Is there any meal in the restaurant which anybody do not eat it? If so,

please write the names. At the end of the activity, students are introduced to the definition of the concept of function as: "*If, A ≠ Ø, a function which is given in set diagrams, denoted by f: A→B or f: x → y. It is written y = f(x). x Є A and y = f(x) Є B.*"

This definition is based on the Bourbaki approach. The core idea of the Bourbaki's definition of function is correspondence of two sets as for each element in the domain there is exactly one element in the co domain (Vinner, 1983). However, it is so abstract to students that they easily forget or tend to ignore it (Vinner & Dreyfus, 1989). Therefore, it would be ineffective to first introduce students to functions as object, for instance sets of ordered pairs. Instead, "a more dynamic definition, such as the definition of function as a dependence relation defined by a rule, would be more appropriate for beginning secondary instruction about functions" (Wilson, 1991, p. 11).

Likely, for that reason, the concept of function is introduced to students in the textbook mediated an activity. By this way, it is aimed that students could conceptualize it in a more meaningful way than just memorizing; because after defining the function concept, various examples of functions are given. Most of these examples are presented in different representations of the function concept such as set of ordered pairs, set correspondence diagrams, tables, equations and graphs. The results of the study, on the other hand, show that even tried to give students the concept of function with different activities, due to the definition is still are the same meaning, students could not make sense of it. So, since industrial vocational high schools have enough technical equipment such as computers; internet networks etc. teachers may make those advantages a reality by conducting the multiple representations of functions with this technical equipment.

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