Auction Theory: Bidder’s Perspective in a Public Out-Cry English Auction

Jagannath Aryal, Don Kulasiri and Garth A. Carnaby

Abstract—This paper provides an overview of auction theory literature. We present a general review on literature of various auctions and focus ourselves specifically on an English auction. We are interested in modelling bidder’s behavior in an English auction environment. And hence, we present an overview of the New Zealand wool auction followed by a model that would describe a bidder’s decision making behavior from the New Zealand wool auction. The mathematical assumptions in an English auction environment are demonstrated from the perspective of the New Zealand wool auction.

Keywords—Bidder, English auction, New Zealand, Wool

I. INTRODUCTION

This paper aims to present an overview of auction theory literature. We begin with a brief history of the auction followed by the types of auctions practiced nowadays. In the second section, we present the review of literature on auction theory highlighting the English auction environment. The third section deals with a brief introduction of the New Zealand wool auction and its relation to the New Zealand Wool Industry (NZWI). In the fourth section, the mathematical assumptions in an English auction environment are demonstrated taking into account the New Zealand wool auction. Further, an illustration of a bidder’s decision making behavior in the New Zealand wool auction is presented in a model. Finally, we summarize the paper and propose future work.

Literally, the word auction is derived from ‘augere’ meaning “to increase”. When we go to the history of auction we can see the practice was started in Babylon from 500 BC [1]. In the modern world, auctions are used primarily for transactions of goods and services. Auctions help in a way to form the price and make a contribution of empirical significance on the exchange of ownership of goods. The study of auction is interesting to the traders because the value of the item is uncertain in an auction. Further, the parties involved (buyer and seller) do not know the exact information of the item. However, among the two parties one is much more familiar with the seller) do not know the exact information of the item. However, an established fact is that the uncertainty of the value of the items under transaction is the beauty of the auction which allows competitive bidding. The areas of transactions where auctions allow theory and its implications to be employed are: houses, cars, agricultural products, art and antiques. Further, the transaction range is becoming wider with the advent of Information and Communication Technology (ICT), for example, in general e-commerce and more specifically is eBay and trademe online auctions. There are four main types of auctions in practice; English, Dutch, first-price sealed-bid and second-price sealed-bid [9]. These auctions are practiced elsewhere for the transaction purpose of goods and services. English auction methods have been used in trading the arts and antiques, the Dutch auction is being used in the trading of cut flowers in the Netherlands, fish in Israel and tobacco in Canada. First-price sealed-bid auctions are primarily used by governments in tenders for procurement. On the other hand, second-price sealed-bid auctions have significant theoretical importance; however, they are seldom used in practice.

Though the practices of auction principles have been used since the Babylonian era (193 A.D.), attention became widespread from 1996 once William Vickery was awarded the Nobel Prize in Economics for his contribution to auction theory [1]. The paper by Vickery entitled “Counterspeculation, Auctions, and Competitive Sealed Tenders” [8] is the widely recommended paper to read by auction theorists [1], [4]. The paper developed some special cases of Revenue Equivalence Theorem (RET). The second price sealed-bid auction is sometimes called the Vickery auction after his name. The English auction is widely used, and the New Zealand wool auction is an example of public out-cry English auction.

II. LITERATURE FOCUSING ON VARIOUS AUCTIONS

A. English Auction

English auctions are the ascending type auction where the auctioneer starts from a minimum price. In an English auction, the auctioneer begins by asking for bids at a low price and then gradually raises the price until only one willing buyer remains. The price increases along with the bidders bidding. Once, only one bidder is there to bid for a certain type of sale item he or she becomes the owner of that item. All the bidders who want to bid
in an English auction environment have their own value for the item. This independent-private-value (IPV) is not known to other bidders. On the other hand, they would be in a common value (CV) environment if the bidders have common information of the item to be auctioned. Further, CV is possible without having common information once the bidders enter the real auction environment and know the value of other bidders. In the real-world auction environment, CV is known to all Participating bidders. Still their respective independent-private-value towards the item is unknown. We can assume that IPV and CV follow some probability distribution. The real-world auction situations are likely to contain aspects of both models simultaneously. The IPV and CV are identically and independently drawn values from a common probability distribution. The IPV and CV of one bidder are statistically independent to other bidders.

Among the various literature on auction theory, [1]-[16] are relevant to this study. The references [1]-[7], covered many aspects of auction. For example, [1] is a collection of journal articles written from 1999 to 2003 and published in various journals. This collection of articles is an extraordinary presentation of auction theory in a non-technical manner. In other words, this book is a thorough review of auction theory. On the other hand, [2] focuses on how to put auction theory to work. Milgrom presented the mathematical overview, in particular the tools of ordinary demand theory that includes the IPV and CV of one bidder. 

III. NEW ZEALAND WOOL AUCTION: AN ENGLISH PUBLIC OUT-CRY AUCTION

The New Zealand Wool Industry (NZWI) is contributing over a billion New Zealand dollars per year to the New Zealand economy. Being a major contributor to the economy, there is a high significance of the study of wool trading systems like auctions in New Zealand. The New Zealand wool auction is a public out-cry auction which has been running for the last 150 years. The New Zealand wool auction is a selling avenue of New Zealand Wool Clip (NZWC). A total of 60% of NZWC is traded via auction. The auction runs in the English auction principle. The procedure of auction operation and participation of the bidder is similar to the online English auction. However, the difference is that the bidders’ are physically present there to make a bid for particular New Zealand wool clip of interest to their business. The NZWC is produced through the combined efforts of some 20,000 individual farmers. During any one twelve month period, each farmer produces a variety of wool types which come forward at times that are dictated by weather and necessary farming practices. There are over 3000 types of wool produced in New Zealand. This reflects also the different breeds, age, and place of origin on the sheep’s body. Wool is further separated and differentiated by various types of possible contamination from vegetable matter to cotts. It is also produced at different locations throughout the country. The diversity inherited in the NZWC resembles a complex system. Being diverse and complex in nature, the bidders’ in the wool auction follow the traditional ways of bidding instead of any automated system. The live scenario of the New Zealand wool auction shows that though the bidding procedure is traditional, the bidders are getting real time feedback via their offices on some major factors like currency fluctuations during the hours. An example of the Christchurch wool auction centre, the only centre in the South island of New Zealand, shows that around forty bidders are registered for the bidding purpose. However,
all of the registered bidders are not actively involved in buying the wool via auction. This shows that the number of bidders during the auction hours is uncertain and hence it follows the independent –private- value (IPV) model and common value (CV) model like in other English auction environments. However, in practice, the major market is controlled by quite a few exporters. Taking the whole New Zealand wool export market into perspective, the statistics show that New Zealand exports 20% of the world wool market [13]. The top ten export destinations of NZWC are shown in Table I.

IV. MATHEMATICAL ASSUMPTIONS IN AN ENGLISH AUCTION: BIDDER’S PERSPECTIVE FROM NEW ZEALAND WOOL AUCTION

In any auction environment, we consider that Revenue Equivalence Theorem (RET) has a significant role. RET in general gives the message that the seller can expect equal profits on average from all the standard as well as non-standard types of auctions [1]. As RET is a fundamental basis for auction theory, we can assume that if the bidders are risk neutral the seller would not benefit from such risk neutral bidders. However, on the other hand, if the bidders are risk averse the seller would expect more benefit from such bidders. The risk averse bidders are more volatile than the risk neutral bidders. Further, as discussed in section II, every bidder may have to face two models simultaneously during the time of bidding.

TABLE I
TOP TEN EXPORT DESTINATION OF NEW ZEALAND WOOL CLIP

<table>
<thead>
<tr>
<th>Country</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>25</td>
</tr>
<tr>
<td>UK</td>
<td>13</td>
</tr>
<tr>
<td>India</td>
<td>10</td>
</tr>
<tr>
<td>Italy</td>
<td>9</td>
</tr>
<tr>
<td>Belgium</td>
<td>8</td>
</tr>
<tr>
<td>Germany</td>
<td>6</td>
</tr>
<tr>
<td>Australia</td>
<td>5</td>
</tr>
<tr>
<td>Japan</td>
<td>4</td>
</tr>
<tr>
<td>Turkey</td>
<td>3</td>
</tr>
<tr>
<td>USA</td>
<td>3</td>
</tr>
</tbody>
</table>

follow two models simultaneously during the time of bidding.

In this paper, we try to follow as closely as possible the standard notation of auction theory papers. Among the various assumptions, we would like to present two assumptions which are relevant to the New Zealand wool auction. The first one is whether RET holds true in the case of the uncertain number of bidders in an English auction environment. And, secondly, what the expected price of the bidder in any English auction environment is.

The suitability of RET in the case of uncertain numbers of bidders which is relevant to the New Zealand wool auction is presented. We follow the proof based on [3], actual bidder \( i \in A \) and let \( p_n \) denote the probability that any participating bidder assigns to the event that he is facing \( n \) other bidders. Thus, bidder \( i \) assigns probability \( p_n \) that the number of actual bidder is \( n + 1 \). The probabilities \( p_n \) do not depend on the identity of the bidder or on his value. It is also important that the set of actual bidders does not depend on the realized values. As long as bidders hold the same beliefs about the likelihood of meeting different numbers of rivals in an auction environment, the proposition holds that the expected payment of a bidder with value zero is zero, and yields the same expected revenue to the seller provided the values drawn are independently and identically distributed.

Let us consider a standard auction \( a \) and a symmetric and increasing equilibrium \( \beta \) of the auction. It should be noted here that the bidders are unsure about the number of rivals they have to face, \( \beta \) does not depend on \( n \). Let us assume that the expected payoff of a bidder with value \( x \) who bids \( \beta(x) \)-strategic bid- instead of the equilibrium bid \( \beta(x) \). The probability that he faces \( n \) other bidders is \( p_n \). In this case, the bidder wins if \( Y^{(x)}_1 \), the highest of \( n \) values drawn from \( F \), is less than \( x \) and the probability of this event is \( G^{(x)}(z) = F(z)^n \). The overall probability that he will win when he bids \( \beta(z) \) is therefore,

\[
G(z) = \sum_{k=0}^{n-1} p_k G^{(x)}(z) \tag{1}
\]

His expected payoff from bidding \( \beta(z) \) when his value is \( x \) is then

\[
\prod^-(z,x) = G(z)x - m^a(z) \tag{2}
\]

This shows that in the situation of uncertain number of bidders RET also holds true.

Now, we would like to present the expected price of the bidder in an English auction environment. Let us take a case of three bidders having actual private types, \( x_1 = a, x_2 = b, x_3 = c \), respectively with \( a < b < c \), after [4]. The implicit assumption is that the price starts at zero and rises continuously. The first player to drop out is player 1 at price \( p_1 \) such that:

\[
E[V_1 | x_1 = a = X_2 = X_3] = p_1 \tag{3}
\]

Prior to bidder 1 dropping out, the expected values of bidders 2 and 3 were given, respectively, by:

\[
E[V_2 | x_2 = b = X_1 = X_3] \tag{4}
\]

and

\[
E[V_3 | x_3 = c = X_1 = X_2]. \tag{5}
\]

After bidder 1 drops out, these expected values are revised in the following way:
Bidder 3 wins the auction at price $p_2$.

If we generalize for $n$ players, the expected price in an English auction environment is,

$$ P^n = E[V_i | X_i = b; V_j | X_j = c; \ldots; V_k | X_k = a; \ldots | X_i = x > Y_j ] $$

At auction, the price keeps rising until the final price $p_2$ and the winner is determined as follows:

$$ E[V_i | X_i = b = X_p, p_1] = p_2 $$

The value of the object to player 3 prior to knowing $p_2$ is

$$ E[V_i | X_3 = c = X, p_1] $$

**Price of the wool in any auction**

Price of the wool in any auction is a function of independent private values and common values of the bidders. This study will help us to make an extension of the functioning model that will help the bidders to make efficient decisions in the wool auction environment. Such semi-automatic tools will contribute to high returns in wool trading.

**V. CONCLUSION**

Auction theory with a highlight of an English auction environment is presented. A model showing the decision making behavior of bidders from the New Zealand wool auction shows that the price of wool type in an auction is the function of independent private values and common values of the bidders. This study will help us to make an extension of the functioning model that will help the bidders to make efficient decisions in the wool auction environment. Such semi-automatic tools will contribute to high returns in wool trading.

**ACKNOWLEDGMENT**

We would like to acknowledge New Zealand Wool Services International (NZWSI) and Malcolm Ching for the discussions during the manuscript preparation.

**REFERENCES**


Fig. 1 A model showing the decision making behaviors of a bidder in the New Zealand wool auction environment.

Jagannath Aryal received his Professional Masters Degree in Geo-informatics from International Institute for Geo-information and Earth Observation (ITC), The Netherlands (2003), a research M.Sc. Degree in image processing from University of Otago, New Zealand (2006). Currently, Mr. Aryal is pursuing his Ph D in Lincoln university. Mr. Aryal became a student member (SM) of Society for Industrial and Applied Mathematics (SIAM) in 2006. His current research interests include application of mathematics and computational modelling.

D. Kulasiri received his Masters Degree (1988) and a Ph D (1990) from Virginia Tech, USA. He is currently a Professor of Computational Modelling and Simulation at the Lincoln University, New Zealand. Prof. Kulasiri is the head of the Centre of Advanced Computational Solutions (CACS) in Lincoln University, New Zealand. Prof. Kulasiri is the member of Modelling and Simulation Society of Australia and New Zealand (MSSAN), American Society for Agricultural Engineers (ASAE), International Society for Computational Biology (ISCB), Society for Industrial and Applied Mathematics (SIAM). Prof. Kulasiri is a visiting professor to the Mathematical Institute, Oxford University UK; a visiting Professor to Institute of Scientific Computing, Technical University of Braunschweig, Germany; Visiting Fellow, Princeton University, New Jersey, USA.

Garth A. Carnaby received his B.S.Hons (1972) from the University of New South Wales, Australia; a Ph.D (1976) from the University of Leeds, UK and an honorary DSc (1995) from De Montfort University based in Leicester, UK. He is active as a leader in a wide range of research projects involving the application of mathematics and physics to the New Zealand wool industry. A fellow of the Royal Society of New Zealand, his main speciality has been the mechanics of fibrous structures. He holds various international patents and has published over 200 research papers and articles.