Abstract
The Australian Learning and Teaching Council funded project, *Building the Culture of Evidence-Based Practice in Teacher Preparation for Mathematics Teaching*, as its title suggests, aims to contribute to the development of a Culture of Evidence-based Mathematics Education for New Teachers (CEMENT) and ultimately for pre-service teacher education generally. Such a culture requires the timely availability of pertinent, valid evidence. This paper reports on the design, development, and piloting of instruments used to collect data from pre-service teachers that will serve as evidence to inform the seven participating universities as they make ongoing changes to their mathematics teacher education provisions.

The conceptual framework underpinning the project was based on that used in the international Teacher Education and Development Study in Mathematics (TEDS-M). The framework includes three domains: Characteristics of Future Teachers (on entry to teacher education programs), Characteristics of Teacher Educators, and Characteristics of Teacher Education Programs. These domains are interrelated yet act independently on outcomes of teacher education programs against a background of policy and practice unique to the place of education. In the Australian context these background factors are similar across states and territories (e.g., National Standards for Teachers and the Australian Curriculum).

Instruments were designed to collect evidence of factors believed to contribute to the quality of programs for pre-service mathematics teachers. The online questionnaire that was devised included items intended to measure respondents’ knowledge of mathematical content, pedagogical content knowledge, and aspects of their beliefs about the nature of mathematics as a discipline and mathematics learning and teaching. For ease of analysis multiple choice items were used and the number of items included was limited by the need to recruit volunteers to complete the questionnaires.

Rasch analysis of pilot data showed that the items work together to measure a single underlying variable, and that pre-service teachers found it much easier to endorse belief statements than to respond appropriately to mathematics content or pedagogical content knowledge items. The relative difficulties of items reflected teaching emphases within the program and have already highlighted areas in which targeted improvements could be made. Subsequent analyses using structural equation models will identify relative influences on pre-service teacher outcomes of factors such as course delivery and structure, lecturer background and students’ characteristics.

Introduction
The project reported in this paper was designed to address the need in Australia for the provision of quality teachers of mathematics at all levels of the pre-tertiary system through evidence-based improvement of pre-service teacher education programs. It is timely in view
of recent developments of national standards for Graduate Teachers (Australian Institute for Teaching and School Leadership [AITSL], 2011b), a national curriculum (Australian Curriculum Assessment and Reporting Authority, 2011), and increased accountability of tertiary institutions in the form of course accreditation standards (AITSL, 2011a). The project will provide tools for universities to monitor their mathematics education courses in terms of students’ outcomes, and devise processes to bring about changes based on the evidence collected. It will provide guidance to teacher education institutions about effective programs and a model for other disciplines to develop accountability tools. Ultimately, the project will contribute to a national culture of evidence-based tertiary pre-service teacher education programs.

Australian institutions involved in the education of pre-service teachers provide a diverse range of courses that are informed largely by anecdotal evidence and research conducted elsewhere. The National Numeracy Review Report (Council of Australian Governments, 2008) pointed out that, “There are many challenges in the pre-service education of mathematics teachers and many areas where the research knowledge is limited” (p. 71). Goos, Smith and Thornton (2008) indicated that there was a need for large-scale, national studies to establish evidence of best practice in mathematics teacher education. Courses vary in their structure, length and mode of delivery, and there is a growing trend towards the use of distance learning technology (Holt & Challis, 2007). Some institutions offer postgraduate courses only while others offer 4-year undergraduate training, and some take account of existing professional experience in schools or training institutions. These pre-service courses attract students including those straight from Year 12, mature age students with no university background, career-change professionals, and para-professionals such as teacher aides seeking to upgrade their qualification (Goos & Callingham, this symposium). Many of these students enter with relatively weak mathematical backgrounds (Mays, 2005).

The project aims to address the lack of coherence in mathematics teacher preparation through provision of national benchmarks for graduate teachers of mathematics. Teacher education institutions will be able to use these benchmarks to assess local evidence to inform course design and monitor ongoing improvement. In this paper we outline the conceptual framework underpinning the project and describe the process of instrument development and the results obtained from a subsequent pilot study involving pre-service teachers.

**Conceptual framework**

The study is underpinned by an adaptation of the conceptual framework used in the international Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto, et al., 2008) shown in Figure 1. The framework for TEDS-M includes three inter-related domains that each independently influences the outcomes of teacher education programs. The three domains; Characteristics of Future Teachers, Characteristics of Teachers Educators, and Characteristics of Education Programs, act within the social, political, cultural and economic context in which the teacher education occurs. This is an important consideration for international studies such as TEDS-M but because Australian universities operate in the context of the same national agendas (National Teacher Standards, National curriculum and National accreditation) this dimension was not included in the framework for this project. Each element of the framework is explained in the sections that follow.
Characteristics of future teachers

The characteristics of future teachers entering teacher education programs is related to selection policies that commonly concern levels of prior mathematics study and/or achievement (Tatto, Lerman, & Novotna, 2010). AITSL (2011a) has proposed that teacher education students be drawn from the top 30% of the population, but the precise meaning of this is unclear. Despite the focus on the mathematics content knowledge of pre-service teachers other characteristics are also likely to be relevant. These include previous professional experience, qualifications, age, gender, location, and attitudes to and beliefs about the discipline of mathematics and what it means to teach and learn the subject. There is evidence that many pre-service teachers, particularly those intending to be primary teachers, commence their teacher education courses with negative or even fearful attitudes to mathematics (Beswick, 2006; Brown, 2009) as well as poor mathematical content knowledge (Brown, 2009; Mays, 2005). Forgasz and Leder (2008) reviewed research that suggest that teacher education, and particularly practicum experiences, can influence the beliefs of pre-service teachers, and Frid and Sparrow (2009) pointed to the need for teacher education to address in a cohesive fashion the various aspects of affect along with content and pedagogical knowledge. Teacher education is thus concerned with influencing those characteristics of future teachers that are subject to change. Many others of the characteristics listed are not subject to change but are, nevertheless, important to consider in studying the relative impacts on teacher education outcomes of pre-service teacher characteristics in comparison with lecturer and institution characteristics. Data concerning them may also be useful in identifying ways in which teacher education courses might usefully be tailored for differing pre-service teacher cohorts.

Characteristics of teacher educators

Just as teachers in school classrooms influence students’ outcomes (P. W. Hill, Rowe, Holmes-Smith, & Russell, 1996) tertiary teachers also influence their students’ outcomes (Biggs, 2003; Tato, 1999). Relevant factors are likely to include the extent and recency of teacher educators’ school teaching experiences, employment status (e.g., tenured, sessional), and mathematics and educational qualifications. Lecturers’ beliefs about the nature of mathematics and how it is best learned and taught as well as about the needs and capacities of pre-service teachers are also relevant. Tato (1999) found that teacher education programs in which lecturers’ held consistent views that were aligned with the curriculum were more successful in influencing the views of pre-service teachers.
Characteristics of teacher education programs

Tatto, Lerman and Novotna (2010) pointed out that although there is evidence that the structure and approach of teacher education programs seem to influence pre-service teachers’ knowledge and practice, it is not clear that these elements impact pre-service teachers’ learning of how to teach specific subjects such as mathematics. The diversity of teacher education program structures has already been noted. Differences relate to factors such as the mode of delivery (e.g. on campus, online, blended), entry level (e.g. undergraduate or post-graduate entry), the number and placement of mathematics units within the course as well as the content of these units, including the extent to which mathematics content is taught in stand-alone units or is integrated with pedagogy, and who teaches pre-service mathematics units (e.g., mathematicians or mathematics educators). Institutional factors including faculty, school and campus arrangements and the ways in which these are managed are also relevant. We know that pre-service teachers greatly value their time in schools (Ashman & McBain, in press; Beswick, 2006; Richardson, 1996) and there is evidence that experience in schools can influence pre-service teachers’ beliefs (Forgasz & Leder, 2008). The length, nature and placement of practicum experiences in the course and in relation to mathematics units are, therefore, also likely to be important.

Pre-service teacher outcomes

It is recognised that teaching mathematics requires much more than simply knowledge of the relevant content. Shulman’s (1987) identification of seven knowledge types for teaching in general has had a lasting impact on the conceptualisation of the knowledge required to teach mathematics. Shulman considered it important that teachers have content knowledge, general pedagogical content knowledge, pedagogical content knowledge (PCK), knowledge of students as learners, knowledge of education contexts, and knowledge of the ends and purposes of education. The knowledge required by teachers of mathematics has received considerable attention since then with Ma (1999) describing “profound understanding of fundamental mathematics and Ball and colleagues (e.g., Ball & Bass, 2000; Ball, Thames, & Phelps, 2008; H. C. Hill, Rowan, & Ball, 2005) analysing what they have described as “mathematical knowledge for teaching”. Beswick (2011) argued from a theoretical perspective that beliefs are in fact a subset of knowledge differentiated from that which is commonly called knowledge by the extent of consensus about their veracity. Beliefs were therefore included in the conceptualisation of teacher knowledge that underpins this project.

Attempts to measure teachers’ knowledge have progressed from early attempts that used the number and nature of mathematics courses studied or years of teaching experience as proxies for knowledge (Mewborn, 2001) to approaches that have built on increasingly sophisticated understandings the kinds of knowledge required. These have included pen and paper instruments of various kinds that have attempted to measure one or more of Shulman’s knowledge types (H. C. Hill, Sleep, Lewis, & Ball, 2007). Measuring PCK has presented the greatest challenge with attempts to measure it including intensive observations and analyses of classroom interaction (Ball & Bass, 2000) and detailed interviews requiring teachers to comment on lesson plans (Zhou, Peverly, & Xin, 2006). Beswick, Callingham and Watson (2011) used a written profile aimed at assessing all of Shulman’s knowledge types in the context of mathematics. They attempted to measure PCK by having teachers suggest anticipated correct and incorrect responses to mathematical problems and to indicate how they would use the problems in their mathematics classrooms.

In this study mathematics content knowledge, PCK for mathematics teaching and beliefs were considered most relevant. The ways in which these were conceptualised and operationalised are described in the sections on instrument development that follow.

Instrument development
Instruments used in the project include questionnaires and interviews. The focus in this paper is on the development of the questionnaires and pilot data obtained from their use. The questionnaires were designed to measure the mathematical content knowledge, PCK and beliefs of primary and secondary pre-service teachers. Different measures, with respect to content knowledge and PCK, were considered appropriate for these groups because of the differing mathematical backgrounds of prospective primary and secondary teachers. The belief items related to views of the discipline and mathematics teaching and learning that were not specific to the level of schooling and hence these items were the same in both the primary and secondary instruments. The decision to focus on content knowledge and PCK rather than others of Shulman’s (1987) knowledge types was related to the mathematics specific nature of these aspects, a degree of overlap between the way that we conceptualised PCK and aspects of knowledge of students as learners, as well as the constraints inherent in a single questionnaire to be completed by volunteers. The need to collect and analyse data from large numbers of pre-service teachers meant that an online format using multiple choice items was appropriate. It was also agreed that a respondents could not be expected to spend more than 45 minutes completing the entire questionnaire. Qualtrics (http://www.qualtrics.com/) was used to deliver a link to the questionnaire to each participant.

The process of questionnaire development was collaborative and began with a face-to-face 2-day meeting of the team. The process by which the instrument was constructed is described in the following sections. It drew upon the collective expertise of the nine member research team all of whom had worked in pre-service mathematics education for a number of years with several having researched and published in the area (e.g., Beswick, 2006; Goos, et al., 2008) including in the nature and development of beliefs and knowledge for mathematics teaching (e.g., Beswick, et al., 2011; Chick, Pham, & Baker, 2006). and was designed to ensure its content validity (Burns, 2000). Construct validity was established using Rasch analysis of the pilot study data.

Content knowledge
Knowing mathematics content was taken to mean more than being able to perform calculations or recall facts and definitions but rather to incorporate understanding of underlying concepts. Appropriate understanding of mathematical content is characterised by the ability to provide mathematical rationales for procedures, identifying connections among mathematical processes and ideas, and to justify particular choices of methods (Ma, 1999). Even when pre-service teachers are competent users of computational algorithms, such conceptual understanding may be absent (Lubinski & Otto, 2004; Mewborn, 2001).

The mathematics content to be covered in the questionnaires was defined by the Australian Curriculum: Mathematics (ACARA, 2011) and aimed to provide coverage of its three strands noting that the Number and Algebra (N & A) strand included more content than the other two strands. The three content strands are N & A, Measurement and Geometry (G & M), and Statistics and Probability (S & P). Members of the project team were assigned the task of writing or adapting items that would assess content in these strands at primary and secondary levels. Each team member prepared four items of which two dealt with content from the N & A strand and one each covered content from the G & M and S & P strands. Equal numbers of primary and secondary items were created where primary and secondary were defined broadly to accommodate differences in these designations among jurisdictions and to include content that could reasonably be expected of all pre-service teachers intending to teach at these levels. Specifically primary was defined as including Years Foundation to 7 and secondary as from Year 8 to Year 10 but with no calculus.

Subsequent to the initial meeting the items were collated and distributed to the entire team for consideration. A teleconference was used to discuss each of the items, suggest
amendments, and to select a subset that would be used on the questionnaires and others
that would be retained in a pool of items for possible future use.

Pedagogical content knowledge
Shulman (1987) defined PCK as “that special amalgam of content and pedagogy that is
uniquely the province of teachers, their own special form of professional understanding”. It
includes knowledge of a range of representations of ideas that are likely to be helpful, and of
difficulties that students are likely to encounter (Ball & Bass, 2000) and entails Ma’s (1999)
notion of profound understanding of fundamental mathematics. Chick, Pham and Baker
(2006) devised a framework for examining mathematical PCK. It comprises three categories
that illustrate the varied extent to which content and pedagogical content may be mixed. The
first is “clearly PCK” in which content and pedagogical knowledge are inextricably linked
such as in knowing about student misconceptions or appropriate teaching strategies. In the
second category, “content knowledge in a pedagogical context”, content knowledge is used
in a pedagogical context such as when unpacking a mathematical concept and identifying
relevant connections among concepts that could be useful in teaching. The third category
“pedagogical knowledge in a content context” is the use of pedagogical knowledge in
specific context of mathematics teaching. It would be used, for example, when setting
objectives for a mathematics lesson or managing a mathematics classroom activity.

Devising items to measure PCK was arguably the most difficult part of the questionnaire
development. As an initial step, the team brainstormed elements that comprise PCK. The
results are shown in Figure 2. Following extensive discussion and tentative attempts to
suggest examples of items that fitted various categories it was decided that the PCK items
for the questionnaire would be drawn from each of the following categories: (1)
analysing/anticipating/diagnosing student thinking, (2) constructing/choosing tasks/tools for
teaching, (3) knowledge of representations, and (4) explaining mathematical concepts. As
for content knowledge, there needed to be separate pools of items suitable for primary and
secondary pre-service teachers.

Each team member was tasked with writing an item in each of the four categories with two
designated primary and two secondary in the same way as for content items. The PCK items
were also collated and distributed and discussed and debated in considerable detail in the
subsequent teleconference and several were modified as a result.

Beliefs
Constraints inherent in a 45 minute questionnaire to measure content knowledge, PCK and
beliefs meant that this section of the questionnaire comprised just ten items. Of these, nine
were statements requiring respondents to indicate the extent of their agreement on 5-point
Likert scales from “1” indicating Strongly Disagree to “5” for Strongly Agree. These items
were drawn from those used in previous studies (e.g., Van Zoest, Jones, & Thornton, 1994)
and chosen so that three related to each of beliefs about the nature of mathematics, beliefs
about mathematics teaching, and beliefs about mathematics learning. The tenth item asked
pre-service teachers to rate on a similar 5-point scale their confidence to teach mathematics
at the grade levels that they would be qualified to teach.
Pilot study

Participants
Fifty five of the 155 primary pre-service teachers enrolled in either of two mathematics units offered in the summer semester in fully online mode at one university completed the online pilot questionnaire. One of the units was the second of two compulsory mathematics curriculum units in the Bachelor of Education program and the other was an elective unit designed to facilitate the development of the pre-service teachers' personal understanding of fundamental mathematics. Approximately 10% of the pre-service teachers were catching up with the course because they had failed an earlier mathematics curriculum unit or had not achieved and acceptable score on a mathematics competency test. Most were approaching the end of their course.

Instrument
The questionnaire, developed as described in previous sections, comprised a total of 46 items: 13 multiple choice items designed to assess mathematical content knowledge, 23 multiple choice item designed to measure PCK, 9 Likert scale items to assess beliefs, and 1 item providing an indication of the pre-service teachers’ perceived preparedness to teach mathematics.

Data analysis
For the pilot study the multiple choice items were coded correct or incorrect. More subtle discrimination among the various response options was used beyond the pilot study. Aspects of the process of assigning scores to various choices is discussed in Chick (this symposium), and initial results using these scores in Callingham and Beswick (this conference).

Rasch analysis of all 46 items was conducted using Winsteps (Linacre, 2011) in order to examine the extent to which the items worked together to measure an underlying construct and, if so, to compare the relative difficulties of the items and item types. The Rasch analysis
was evaluated by a consideration of infit mean square values for both items (INMSQ_I) and persons (INMSQ_P) (Bond & Fox, 2007). Generally accepted levels of fit lie between 0.77 and 1.3 logits (Keeves & Alagumalai, 1999) and have an ideal value of 1.0 logit. These statistics were available from the Winsteps output. The software also produces a Wright map of the variable, showing both items and persons on the same measurement scale. This output provides a visual picture of the relative difficulty of every item and relative ability of every person.

Results and discussion

The overall fit values were satisfactory for both persons and items (MNSQ_P = 0.98; MNSQ_I = 0.96) suggesting that the items from the three domains of beliefs, MCK, and PCK did provide a measure of a single unidimensional construct which we called Teacher Knowledge. In addition, the coherence of the items was checked by considering the fit of individual items. This test provides a more rigorous test of unidimensionality. There was no item misfit confirming the presence of an overall construct of Teacher Knowledge.

Figure 3 shows the variable or Wright map for all items, separated according to item type (PCK, mathematical content knowledge (MCK), and beliefs (BELF)). It is apparent that with the exception of one PCK item that related to proportional reasoning, the pre-service teachers found the PCK and MCK items difficult and the beliefs items easy. That is, it was much easier for the pre-service teachers to endorse the belief statements than to provide correct responses to the content knowledge and PCK items.

The relative difficulties of some of the content and PCK items were unexpected and have already prompted reflection on the mathematics curriculum units offered in the course in which the pilot study participants were enrolled. For example, the easiest PCK item required the pre-service teachers to choose the most appropriate representation of a proportional reasoning task. The item is shown in Figure 4. The development of proportional reasoning is recognised as a crucial element of the mathematics curriculum in the middle years of school
and has been described as “a cornerstone of higher mathematics” (Lamon, 1993, p. 41). It is inherent in understanding of topics including rational numbers with which pre-service teachers have been shown to struggle (Mewborn, 2001). Because of these facts, proportional reasoning had received considerable attention in the B. Ed. curriculum. These data suggest that this focus has been helpful in assisting pre-service teachers to develop their capacity to teach these ideas. Interestingly the content items that required respondents to identify which of four diagrams showed \( \frac{3}{4} \) shaded and to convert between Australian and Brunei dollars by first converting to British pounds using two graphical representations of the proportional relationships were among the easiest of the content items although these were, like all of the content items, still difficult and considerably more difficult than the particular PCK item shown in Figure 4.

One of the most difficult PCK items is shown in Figure 5. It involved choosing the most appropriate representation for developing young children’s ability to subitise. Given that subitising is defined in the item stem and that the topic was taught in the first mathematics curriculum unit of the course, the difficulty of this item was surprising.

![Figure 4. The least difficult PCK item.](image)

Difficult content items included those related to geometric and measurement concepts that had received relatively little attention in the course. The data suggest that it is unsafe to assume that these areas pose fewer difficulties than those that have been the focus of the course to date.
The ease with which the pre-service teachers were able to endorse the belief statements might, at first glance be encouraging because all were statements that accorded with the constructivist beliefs of the mathematics educators that underpinned the course. It has, however, long been recognised that teachers tend to teach as they were taught (Ball, 1990) even though their teacher education programs appear to influence them to adopt beliefs that are consistent with more progressive teaching approaches. Frid and Sparrow's (2009) assertion that teacher education needs to bring together cohesively confidence and other aspects of affect as well as content and pedagogical knowledge seems apt. These initial data could be indicative of a situation in which pre-service teachers quite genuinely adopt the rhetoric of beliefs that align with those of their lecturers but that their courses allow neither the time nor opportunity for them to reflect sufficiently on the meaning and implications of the beliefs that they endorse for them to become meaningfully integrated with their existing beliefs. Alternatively beginning teachers might be constrained from teaching consistently with their beliefs by insufficient content knowledge and PCK. Interview data will allow these possibilities to be explored. It is clear from these data that for these pre-service teachers agreeing with the kinds of statements commonly associated with desirable approaches to mathematics teaching and expressing confidence in one's ability to teach the subject is not necessarily indicative of the levels of content knowledge and PCK that might reasonably suggest competence.
Conclusion

The pilot study showed that the items worked together to measure a single underlying construct, Teacher knowledge. Refinements of the items and the scoring of responses will allow more detailed examinations of the outcomes of preservice teacher mathematics education and the factors that influence it.

Considered overall, these data confirm that pre-service primary teachers struggle with both mathematical content knowledge and PCK and could be seen as vindicating the attention to content knowledge and pedagogical skill afforded by regulatory bodies such as AITSL (2011a, 2011b) as well as in courses such as the one from which these participants were drawn. However, they also highlight the need more effectively to challenge pre-service teachers to justify the beliefs that they easily endorse and to consider more deeply their implications for practice.

These data have already prompted important reconsideration of the content and delivery of mathematics education in the course in which the questionnaire was piloted. Ongoing and more detailed analyses using refined items and scoring schemes are likely to provide further impetus for ongoing improvement that is grounded in evidence. The data raise but leave unanswered important questions including: How do we develop PCK in pre-service teachers? What knowledge to mathematics educators need to "teach" PCK? How can we assist pre-service teachers to interrogate their beliefs? What roles do and should our own beliefs play? To what extent might a similar approach to evidence-based improvement be useful in other subject areas?

References


