# Full-space diversity and full-rate distributed space–time block codes for amplify-and-forward relaying networks

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**Abstract:** In this study, the authors propose a full-rate distributed space–time block code (D-STBC) that can operate on an arbitrary number of available relaying nodes which generates minimal network overhead when incorporating additional relaying nodes into transmission stream. The proposed D-STBC also does not require any coordination among participating relaying nodes to share any information among each other. The design of the code allows for single-symbol decoding complexity which is not always possible in other D-STBC's when the number of relaying nodes increases. The proposed D-STBC requires a minimum of two participating relaying nodes and because of anonymity among the relaying nodes to each the addition or removal of relaying nodes can be done with ease without disrupting the transmission. They present empirical results in this study to support these claims of the proposed code.

## 1 Introduction

Distributed space–time block codes (D-STBCs) have been extensively studied for their ability to exploit the diversity inherent in a wireless communication channel in order to improve signal quality [1–4]. The D-STBC is a cooperative (relaying) communication scheme originally proposed in [5], which uses client nodes as relaying nodes to emulate a virtual multiple-input–multiple-output (MIMO) channel. The space-diversity improves as the number of relaying nodes increases as a result of more communication paths that become available. In most D-STBCs, the trade off in code design is between a decrease in the code rate and the need to retain the single-symbol decoding complexity at the receiver when more relaying nodes are utilised. A second constraint is how well the cooperative network deals with the addition and removal of relaying nodes and what this entails for the D-STBC encoding and decoding processes operating on each relaying node and the receiver. This is important as incorporating more relaying nodes can improve the diversity gain, while treating all relaying nodes as permanent connections is impractical as these nodes might be clients' apparatus and may not always be available. Most D-STBCs require additional network overhead when additional relaying nodes become available and there is a need to pass along all information necessary to generate the encoding matrices at their respective relaying nodes. If all permutations of these encoding matrices cannot be stored locally on all relaying nodes, then this overhead on the network could potentially degrade the expected performance gain of the utilising D-STBC.

### *1.1 Prior work*

The overall interest in this field of research lies in the construction of a high-rate D-STBC that have the ability to utilise any arbitrary number of relaying nodes while maintaining a single-symbol decodability at the destination [6, 7]. In [8], the authors proposed the use of a row-monomial D-STBC, which is a subset of D-STBCs that has the attribute of uncorrelated noise in the different paths at the receiver (diagonal noise covariance matrix). The systematic construction of this D-STBC results in a code rate equal to or less than $2/(2+N)$, where $N$ denotes the number of relaying nodes. In [9], a new class of D-STBCs called semi-orthogonal precoded distributed single-symbol decodable STBC was proposed and has the advantage of performing precoding on the information symbols before transmitting it to all the relaying nodes which has the potential of nearly doubling the code rate. However, the bandwidth utilisation (code rate) of both these methods [8, 9] decreases dramatically as the number of relaying nodes is increased. Therefore, although these codes are designed for an arbitrary number of relaying nodes, it may be preferable to implement them only for a network with just a few relaying nodes. In [10], a method is proposed that operates on any number of relaying nodes and has single-symbol decodability at the receiver. Although, the offered code rate (one-fourth at most) is independent of the number of relaying nodes (unlike most known D-STBCs), the method has a high decoding delay as the required transmission time increases exponentially with the number of relaying nodes. In [11], an adjustable full-rate STBC matrix is designed for relaying networks that rely on information provided by a feedback channel to adjust the code. This creates additional network overhead that inherently reduces the diversity gain of the network.

The distributed ABBA code was proposed in [12–14], as a generalised ABBA code (GABBA) [15] that was adapted to operate in a distributed amplify-and-forward (AF) network and offers a full-rate while maintaining single-symbol decodability. The GABBA codes do, however, have some constraints which are: (i) the number of symbols per block ($T$) should be expressible as a power of two and (ii) the number of available relaying nodes ($N$) should be smaller or equal to the number of symbols in each transmitted block ($N \leq T$).

Although all the D-STBC research listed above can accommodate any number of relaying nodes available (note only D-GABBA can offer full coding-rate), they all require the utilised number of relaying nodes to remain permanent throughout the communication. Furthermore, to the best of our knowledge all D-STBC schemes also require the transmission of additional control information overhead for the generation of respective encoding matrices at each relaying node. This overhead as stated earlier can potentially reduce the performance (excess bandwidth utilisation) as additional information must be shared among cooperating nodes.

### *1.2 Our contribution*

We show that the coding scheme that was originally proposed in [16, 17] for conventional point-to-point (P2P) MIMO networks, can be adapted for use in an AF relaying network. We show that the proposed D-STBC can accommodate any number of relaying nodes at full code rate with single-symbol decoding complexity, but without the constraints identified for the D-GABBA code

[14]. The proposed D-STBC's code rate is independent of the number of relaying nodes and therefore does not rely on the number of relaying nodes to be small which is a major advantage over most other D-STBC's. In addition, the proposed D-STBC deploys only two identical encoding matrices at every relaying node. This eliminates the need for additional network overhead to coordinate the generation of encoding matrices at particular relaying nodes. Owing to the simplistic design of the proposed D-STBC, it is not reliant on the number of relaying nodes to remain constant for successful communication. The only restriction for the proposed method is that the receiving node must be equipped with at least two antennas. This constraint is inherited from the original code's design and the reliance on knowledge of the channel state information (CSI) of source–relay and relay–destination links at the relays. This is not seen as a severe limitation as most receivers have two or more antennas in modern designs.

This paper is organised as follows. First, a description of the network model is introduced. Then, we propose the D-STBC outlined above by generalising the MIMO code of [16, 17] for use in AF relaying networks, followed by an analysis of the diversity gain offered by such a network. We then present empirical results to back up our claims, and some concluding remarks are presented in the final section.

*Notations:* Hereafter, non-bold letters, bold lowercase letters and bold capital letters will designate scalars, vectors and matrices, respectively. If $\boldsymbol{A}$ is a matrix, then $\boldsymbol{A}^{\mathbf{H}}$, $\boldsymbol{A}^*$ and $\boldsymbol{A}^{\mathbf{T}}$ denote the Hermitian, the conjugate and the transpose of $\boldsymbol{A}$, respectively. The $V(\boldsymbol{A})$ is a function transforming a matrix $\boldsymbol{A}$ to a vector by concatenating the columns vertically. The operator $\otimes$ is the Kronecker product. A complex Gaussian distribution is denoted by $\mathcal{CN}(\mu, \Gamma^2)$, with corresponding mean $\mu$ and diagonal covariance matrix $\boldsymbol{\Gamma}^2$ with value $\sigma^2$.

## 2 Network model

The network model is comprised of a source $S$, $K$ relaying nodes $R_1, \ldots, R_K$ and a destination $D$ which is depicted in Fig. 1. All nodes are equipped with a single antenna, except the destination node that is equipped with two antennas ($M = 2$). The transmissions of symbols through the network are conducted in two phases: the first phase is when the source broadcasts its information to the relaying nodes. In the second phase, all active relaying nodes amplify-and-forward a linear combination of a scaled version of the received signal and its conjugate. The total transmission power for the entire network is denoted by $P_{\text{total}}$, and is evenly divided between the two phases. It should be noted that by employing this code, the source and the destination are assumed to be completely blind, whereas perfect CSI is assumed at the relaying node.

Thus the received symbols at relaying node $R_k$ is given by

$$\underset{T_1\times 1}{\boldsymbol{y}_{R_k}(i)} = \sqrt{P_1}\,\underset{1\times 1}{h_k(i)}\,\underset{T_1\times 1}{s(i)} + \underset{T_1\times 1}{\boldsymbol{w}_k(i)} \tag{1}$$

where $T_1 = T = 2$ is the signalling period in phase 1, $P_1 = P_{\text{total}}/2$ is the power dedicated for phase 1, $\boldsymbol{y}_{R_k}(i)$ is the received symbol at $R_k$ of transmitted block $i$, $s(i)$ is the block $i$ of symbols transmitted from the source, $h_k(i)$ is the channel coefficients at block $i$ (as all channels here are modelled as a Rayleigh block fading channel) from the source to the relaying node $R_k$ as $h_k \sim \mathcal{CN}(0,\ 1)$ and $\boldsymbol{w}_k(i)$ denoting the noise vector at the relaying node $R_k$ with entries $w_j \sim \mathcal{CN}(0, 1)$.

The network employs a selective AF relaying protocol [5], where it is assumed that only a subset of the available relaying nodes will participate in the relaying phase.

The signal-to-noise ratio (SNR) is used as a criterion to determine whether a relaying node can participate, for example, $\gamma_{R_k} \geq \gamma_{\text{th}}$, where $\gamma_{R_k}$ is the SNR at relaying node $R_k$ and $\gamma_{\text{th}}$ is a threshold that is optimally designed according to the definition of the protocol [5]. To limit the discussion to the proposed code, the choice of $\gamma_{\text{th}}$ is not considered here. Therefore it is assumed that $N$ out of $K$ relaying nodes satisfies the criterion. Without loss of generality, we will refer to the $N$ participating relaying nodes as relaying nodes throughout this paper. The relaying nodes $R_n$, $n \in [1, N]$ and $N \leq K$, have to scale their received signals accordingly as

$$\boldsymbol{u}_{R_n}(i) = \rho_{R_n}\boldsymbol{y}_{R_n}(i) \tag{2}$$

where $\rho_{R_n}$ is the scaling factor at $R_n$ and is computed as

$$\rho_{R_n} = \sqrt{\frac{1}{C_n}}\rho \tag{3}$$

with

$$\rho = \sqrt{\frac{P_2}{P_1 + 1}} \tag{4}$$

The variable $P_2 = P_1/N$ and $C_n$ is computed as

$$C_n = \sum_{m=1}^{2} |g_{mn}|^2 \tag{5}$$

The relaying node then forwards the amplified received symbols to the destination after encoding them according to the D-STBC used (see Section 3). The received symbol at the destination is given as

$$\underset{T_2\times M}{\boldsymbol{Y}_D(i)} = \underset{T_2\times N}{\boldsymbol{X}(i)}\,\underset{N\times M}{\boldsymbol{G}(i)} + \underset{T_2\times M}{\boldsymbol{V}(i)} \tag{6}$$

where $\boldsymbol{Y}_D(i)$ is the received symbols at $D$ of the transmitted block $i$, $T_2 = T_1 = 2$ is the signalling period in phase 2, $\boldsymbol{X}(i)$ is the encoded symbols of block $i$ formed distributively using $\boldsymbol{u}_{R_n}(i)$ by the relaying nodes (see Section 3), $\boldsymbol{G}(i)$ is the channel matrix from the relaying nodes to the destination of block $i$ and $g_{mn} \sim \mathcal{CN}(0,\ 1)$ and $\boldsymbol{V}(i)$ is the noise matrix at the destination with entries $v_{jk} \sim \mathcal{CN}(0,\ 1)$.

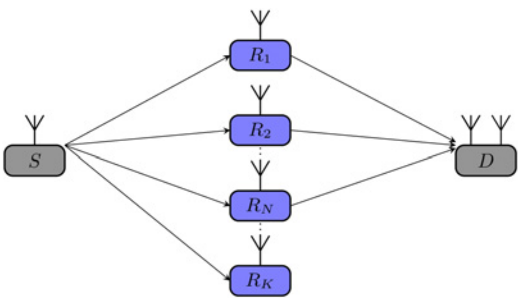


**Fig. 1** *AF relaying network model*

## 3 D-STBC design with joint transmit/receive antenna diversity

In this section, we will discuss how the space–time block coded-joint transmit/receive antenna diversity (STBC-JTRD) proposed in [16, 17] was adapted to operate in an AF relaying network. The resulting distributed scheme (D-STBC-JTRD) can accommodate any number of relaying nodes (with full-space diversity) while retaining a full-rate code with single-symbol complexity decoding. In addition, the adapted code only requires two identical encoding matrices at all relaying nodes, which mitigate the network overhead as no coordinated process is required to generate and distribute different encoding matrices among the relaying nodes.


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### 3.1 Code construction

If the user data is denoted by $s_1$ and $s_2$, omitting the index $i$ (the symbol block index) then without loss of generality, the code of [16, 17] for the case of a two-antenna receiver can be adapted for a distributed scheme as

$$\boldsymbol{X} = \begin{pmatrix} s_1 \\ -s_2^* \end{pmatrix} \otimes \boldsymbol{g}_0^* + \begin{pmatrix} s_2 \\ s_1^* \end{pmatrix} \otimes \boldsymbol{g}_1^* \tag{7}$$

where $\boldsymbol{X}$ is the general code matrix which should be generated, $\boldsymbol{g}_m^* = \begin{bmatrix} g_{m,1} & g_{m,2} & \cdots & g_{m,N} \end{bmatrix}^*$ and the variable $g_{m,n}$ denotes the channel coefficient from the relaying node $R_n$ to the destination antenna $m$, $m \in [1, 2]$.

Thus, each relaying node should encode their received symbols as

$$\boldsymbol{X}_{R_n} = \begin{pmatrix} d_{n,1} \\ -d_{n,2}^* \end{pmatrix} \times g_{1,n}^* + \begin{pmatrix} d_{n,2} \\ d_{n,1}^* \end{pmatrix} \times g_{2,n}^* \tag{8}$$

where $\boldsymbol{X}_{R_n}$ is the encoding output that is computed at each relaying node $R_n$ to generate (7) distributively. The scalars $d_1$ and $d_2$ are computed as

$$\underbrace{\begin{pmatrix} d_{n,1} \\ d_{n,2} \end{pmatrix}}_{\boldsymbol{d}_n} = \frac{h_n^{\mathrm{H}}}{\left\| h_n^{\mathrm{H}} \right\|^2} \times \boldsymbol{u}_{R_n}(i) \tag{9}$$

From (8), it is observed that the encoding matrices within each relaying node $R_n$ is a combination of

$$\boldsymbol{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{10}$$

Both matrices satisfy the condition for a unitary matrix, which is a prerequisite to be used in AF relaying network [18]. The transmission of the symbols from the relaying node $R_n$ to the destination is given as

$$\boldsymbol{t}_{R_n} = \boldsymbol{F}_{1,R_n}^* \boldsymbol{A}_1 \boldsymbol{d}_n + \boldsymbol{F}_{2,R_n}^* \boldsymbol{B}_1 \boldsymbol{d}_n^* \tag{11}$$

with $n = \{1, \ldots, N\}$ and

$$\boldsymbol{F}_{1,R_n}^* = \begin{bmatrix} g_{1,n} & g_{2,n} \\ 0 & 0 \end{bmatrix}^*, \quad \boldsymbol{F}_{2,R_n}^* = \begin{bmatrix} 0 & 0 \\ g_{1,n} & g_{2,n} \end{bmatrix}^* \tag{12}$$

This code uses the same two encoding matrices at all the relaying nodes in a cooperative network to generate the general code matrix in (7). In a conventional D-STBC network, a coordinated process must be used to assist the relaying node $R_n$ to generate its own set of encoding matrices ($\boldsymbol{A}_n$ and $\boldsymbol{B}_n$), whereas for the new proposed scheme we use the same two encoding matrices ($\boldsymbol{A}_1 = \boldsymbol{A}_2 = \cdots = \boldsymbol{A}_N$ and $\boldsymbol{B}_1 = \boldsymbol{B}_2 = \cdots = \boldsymbol{B}_N$). Therefore no network overhead is necessary to convey the control information to arrange which relaying node should generate a symbol using a certain column. In addition, the proposed scheme can add or remove any number of relaying nodes during transmission.

### 3.2 Decoding algorithm

The decoding can be accomplished as

$$\begin{pmatrix} \hat{s}_1 \\ \hat{s}_2 \end{pmatrix} = \begin{pmatrix} y_D^{1,1} + \left(y_D^{2,2}\right)^* \\ y_D^{1,2} - \left(y_D^{2,1}\right)^* \end{pmatrix} \tag{13}$$

where $y_D^{k,l}$ is the element $(k, l)$ of $\boldsymbol{Y}_D$.

*Lemma 1:* The decoding algorithm provided in [16, 17] is applicable to this distributed scheme proposed in (11).

*Proof:* Substituting (1) and (2) into (9) yields

$$\begin{pmatrix} d_{n,1} \\ d_{n,2} \end{pmatrix} = \rho \sqrt{\frac{P_1}{C_n}} \boldsymbol{s} + \rho \frac{1}{\sqrt{C_n}} \frac{h_n^{\mathrm{H}}}{\left\| h_n^{\mathrm{H}} \right\|^2} \boldsymbol{w}_n \tag{14}$$

Substituting (14) into (11) yields

$$\boldsymbol{t}_{R_n} = \rho \frac{1}{\sqrt{C_n}} \left( \boldsymbol{f}_{1,R_n}^* \boldsymbol{A}_1 \left( \sqrt{P_1} \boldsymbol{s} + \frac{h_n^{\mathrm{H}}}{\left\| h_n^{\mathrm{H}} \right\|^2} \boldsymbol{w}_n \right) + \boldsymbol{f}_{1,R_n}^* \boldsymbol{B}_1 \left( \sqrt{P_1} \boldsymbol{s} + \underbrace{\frac{h_n^{\mathrm{T}}}{\left\| h_n^{\mathrm{H}} \right\|^2}}_{\alpha_n^*} \boldsymbol{w}_n \right)^* \right) \tag{15}$$

Each relaying node's symbol, $\boldsymbol{t}_{R_n}$, experiences a multi-path Rayleigh block fading channel modelled by the channel matrix $\boldsymbol{G}$ and added them up at the destination. Therefore (6) can be rewritten as

$$\boldsymbol{y}_D^1 = \rho \sum_{n=1}^{N} \frac{\boldsymbol{G}_n^1}{\sqrt{C_n}} \left( \sqrt{P_1} \boldsymbol{s} + \alpha \boldsymbol{w}_n \right) + \boldsymbol{v}^1 \tag{16}$$

$$\boldsymbol{y}_D^2 = \rho \sum_{n=1}^{N} \frac{\boldsymbol{G}_n^2}{\sqrt{C_n}} \left( \sqrt{P_1} \boldsymbol{s}^* + \alpha^* \boldsymbol{w}_n^* \right) + \boldsymbol{v}^2 \tag{17}$$

where superscript denotes here the column of the matrix, for example, $Z^l$ is the $l$th column of the matrix $Z$

$$\boldsymbol{G}_n^1 = \begin{bmatrix} \left|g_{1,n}\right|^2 & g_{1,n} g_{2,n}^* \\ g_{1,n} g_{2,n}^* & \left|g_{2,n}\right|^2 \end{bmatrix}, \quad \boldsymbol{G}_n^2 = \begin{bmatrix} g_{1,n} g_{2,n}^* & \left|g_{1,n}\right|^2 \\ \left|g_{2,n}\right|^2 & -g_{1,n}^* g_{2,n} \end{bmatrix}$$

The former derivation concluded using (13) to the following result

$$\begin{pmatrix} \hat{s}_1 \\ \hat{s}_2 \end{pmatrix} = \rho \sqrt{2CP_1} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \underbrace{\frac{\rho}{\sqrt{C}} \sum_{n=1}^{N} \alpha_n \left( \left|g_{1,n}\right|^2 + \left|g_{2,n}\right|^2 \right) \boldsymbol{w}_n + \begin{bmatrix} v(1,\ 1) + v^*(2,\ 2) \\ v(2,\ 1) - v^*(1,\ 2) \end{bmatrix}}_{\boldsymbol{W}} \tag{18}$$

where $C = \sum_{n=1}^{N} C_n$.

The noise terms in (18), denoted as $\boldsymbol{W}$, have $E[W] = \boldsymbol{0}$ and $\mathrm{Var}[\boldsymbol{W}] = \sigma_W^2 = \left( \sigma_v^2 + \sigma_w^2 \rho^2 C / \left( \sum_{n=1}^{N} \left|h_n\right|^2 \right) \right) I_T$, where $\sigma_w^2 = \sigma_v^2 = 1$ Therefore $\hat{s}$ is still a Gaussian random vector with mean of $\rho s\sqrt{2CP_1}$ and variance of Var[$\boldsymbol{W}$]. Thus, the decoding can be conducted using maximum likelihood (ML) on each received equalised time-slots as

$$\hat{s}_{j,\det} = \arg\min_{s_l \in S}\left\{|\hat{s}_j - \lambda s_l|^2\right\} \tag{19}$$

where $S$ is the symbol constellation used and $\lambda = \rho\sqrt{CP_1}$.

Let us define some definitions for our discussion.

*Definition 1 (single-symbol ML decodable):* A code or a scheme is said to be single-symbol ML decodable, if its ML decoding metric can be written as a sum of several terms, each of which depends at most on one transmitted symbol [19].

*Definition 2 (decoding delay):* This is the time that the destination has to wait, starting from the time of transmission of symbols from the source, before it can start decoding the D-STBC matrix that it receives [10].

The proposed code has single-symbol decoding complexity at the destination as each equalised time-slot shown in (18) is dependent on only one transmitted symbol. In addition, the proposed scheme has a decoding delay of four time-slots which are attributed to the fact that the receiver has to wait for two time-slots on each transmission phase before decoding the two symbols that was transmitted. The proposed code's decoding delay is independent of the number of relaying nodes, unlike the methods proposed in [19, 10]. Lastly, the decoding is structured in such a way as to cancel out more noise terms which improve the coding gain. This gain obtained from cancellation of noise terms can be observed from (18), when compared with a conventional D-STBC employing an AF protocol, namely (6) of [18].

## 4 Diversity analysis

Let us define some definitions for our discussion.

*Definition 3 (space-diversity order (gain) of wireless communications network):* The wireless communications network is said to achieve space-diversity order of $\ell$ if the average error probability $p(\gamma)$ decays as the inverse of the $\ell$th power of SNR ($\gamma$), which can be stated mathematically as

$$p(\gamma) \le b\gamma^{-\ell} \tag{20}$$

where $b > 0$ is a constant independent of $\gamma$.

*Definition 4 (full-space-diversity relaying communications network):* Consider a cooperative (relaying) communications network comprised of a source equipped with $m$ antennas, a destination with $n$ antennas and $R$ single-antenna relaying nodes. It can achieve full-space diversity if its diversity order is equal to [6]

$$\ell = \min(m \times R,\ R \times n) \tag{21}$$

Thus, the diversity order that the network can achieve assuming $N$ relaying nodes is determined by the SNR of the ML estimate in (19) which is expressed as

$$\gamma = \frac{2CP_1\rho^2}{2\sigma_v^2 + 2\sigma_w^2\rho^2 C/\left(\sum_{n=1}^{N}|h_n|^2\right)} \tag{22}$$

If the SNR between the relaying nodes and receiver is high, then according to [20] the following relationship holds

$$2\sigma_v^2 + 2\sigma_w^2\rho^2 C/\left(\sum_{n=1}^{N}|h_n|^2\right) = 2\sigma_w^2\rho^2 C/\left(\sum_{n=1}^{N}|h_n|^2\right) \tag{23}$$

Given (23), the SNR in (22) is approximated by

$$\gamma = \gamma_0\sum_{n=1}^{N}|h_n|^2 \tag{24}$$

where $\gamma_0 = P_1/\sigma_w^2$ is the nominal SNR in the network.

From (24), it can be concluded that the maximum diversity the scheme can achieve is $\ell = N$ as the total SNR is affected by $N$ channel coefficients.

The average error probability ($p(\gamma)$) is defined as

$$p(\gamma,\ u) = E_{\boldsymbol{h},\boldsymbol{g}}\left(Q\left(\sqrt{u\gamma}\right)\right) \tag{25}$$

where $\boldsymbol{h} = [h_1, h_2, \ldots, h_N]^{\mathrm{T}}$, $\boldsymbol{g} = \mathrm{vec}(\boldsymbol{G})$ and $u$ is a constant scalar that is dependent on the size of the constellation.

Using the Craig's formula for the $Q$-function [21], the $p(\gamma, u)$ of (25) can be rewritten as

$$p(\gamma,\ u|\boldsymbol{h}) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{u\gamma}{2\sin^2\theta}\right)\mathrm{d}\theta \tag{26}$$

*Lemma 2:* Assume a random variable $\boldsymbol{a}$, $\boldsymbol{a} \sim \mathcal{CN}(0_{k\times 1},\ \mho)$. Then the moment generating function of $\|\boldsymbol{a}\|^2$ is given by

$$M_{\|\boldsymbol{a}\|^2}(s) = E_{\boldsymbol{a}}\left[\exp\left(s\,\|\boldsymbol{a}\|^2\right)\right] = \prod_{i=1}^{r}\frac{1}{1 - s\lambda_i} \tag{27}$$

where $\{\lambda_1, \ldots, \lambda_r\}$ be the non-zero eigenvalues of $\mho$.

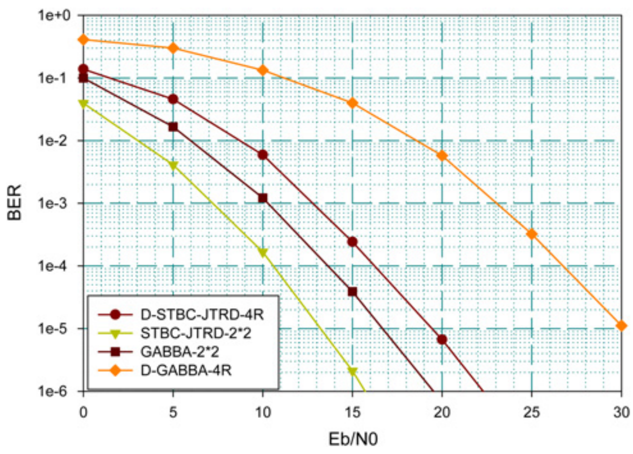


**Fig. 2** *BER of GABBA and STBC-JTRD (P2P and distributed AF network)*




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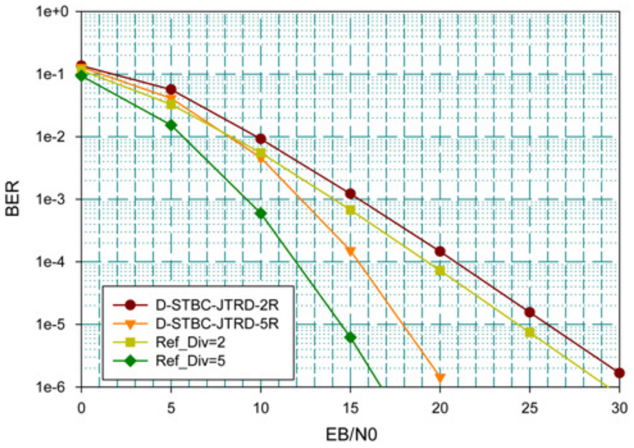


**Fig. 3** *BER of 1 × 2 × 2 and 1 × 5 × 2 D-STBC-JTRD scheme*

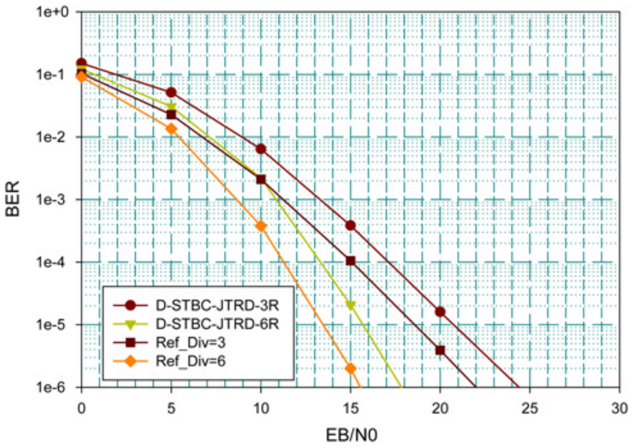


**Fig. 4** *BER of 1 × 3 × 2 and 1 × 6 × 2 D-STBC-JTRD scheme*

*Proof:* Refer to Lemma 1 proof of [21].
Using Lemma 2 in (27), the averaging over $\boldsymbol{h}$ gives

$$\begin{aligned} p(\gamma,\, u) &= \frac{1}{\pi}\int_0^{\pi/2} \prod_{n=1}^{N} \frac{1}{1 + \left(u\gamma_0/2\,\sin^2\,\theta\right)}\,\mathrm{d}\theta \\ &\simeq \frac{1}{\pi}\int_0^{\pi/2} \frac{1}{\left(u\gamma_0/2\,\sin^2\,\theta\right)^N}\,\mathrm{d}\theta \simeq c.\gamma_0^{-N} \end{aligned} \quad (28)$$

From (28), it can be concluded that the space-diversity order of the network $\ell = N$ and according to Definition 2 the network is said to achieve the full-space diversity as $\ell = \min(1 \times N,\ N \times 2) = N$.

## 5 Simulation results and discussion

In this section, we present performance results of the proposed scheme (D-STBC-JTRD) for different number of relaying nodes $N$ to confirm our theoretical results. The quadrature phase shift keying modulation was used and the performance is reported via bit error rate (BER) evaluated over a Rayleigh fading channel. A slow non-selective frequency fading channel was used, which resulted in a fixed fading coefficient over a fixed number of $T$. We investigated the spatial diversity gain by utilising 2, 3, 5 and 6 relaying nodes to illustrate the increase in diversity gain. Higher space-diversity order was achieved while retaining a fixed code rate and single-symbol decoding complexity.

In Fig. 2, the BER is shown for the proposed code compared with the D-GABBA code with both utilising four relaying nodes ($N = 4$) [14]. In addition, a simulation of a MIMO P2P is shown for 2 × 2 GABBA and 2 × 2 STBC-JTRD to compare the performance degradation because of adapting to a distributed AF networks [15–17]. Like our proposed code, the D-GABBA code was chosen as it can offer full-rate for any arbitrary number of relaying nodes. It was observed that the proposed code achieves the full-space diversity, $\ell = \min(1 \times 4,\ 4 \times 2) = 4$ similar to the D-GABBA codes. The added advantage of the proposed code, unlike the D-GABBA code, is there and is no need to adjust the coding scheme which results in reduced network overhead [14].

In Fig. 3, the BER of 1 × 2 × 2 and 1 × 5 × 2 D-STBC-JTRD is simulated to show the space-diversity gain achieved. The BER of $\ell = 2$ and $\ell = 5$-diversity networks was selected for comparison purpose. As discussed in Section 3, it is shown from Fig. 3 that full-space diversity is achieved as the BER curve approximates the reference curves for high SNR.

Similarly, the BER of 1 × 3 × 2 and 1 × 3 × 2 D-STBC-JTRD is simulated in Fig. 4, to show the space-diversity gain achieved when more relaying nodes are available for utilisation.

## 6 Conclusion

This paper proposed a general cooperative network referred to as the D-STBC-JTRD network. It is a scalable network and is able to accommodate varying number of relaying nodes. It shows high efficiency when operating on the AF relaying networks, and has a key feature when employed in the down-link as it assumes no CSI at the destination. The code has been shown to provide full-space-diversity order, while maintaining a full code rate. The code also has the advantage of having a single-symbol decoding complexity and because of the simplistic design requires no additional network overhead when adding or removing relaying nodes.

## 7 References


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