Ocean Engineering 114 (2016) 79–86

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Uncertainty analysis procedure for the ship inclining experiment

Michael D. Woodward [a,*], Martijn van Rijsbergen [b], Keith W. Hutchinson [c], Andrew Scott [d]

[a] Australian Maritime College, University of Tasmania, Launceston, Australia
[b] MARIN, Wageningen, The Netherlands
[c] Babcock International Group Centre for Advanced Industry, Newcastle-upon-Tyne, UK
[d] Maritime & Coastguard Agency, Newcastle-upon-Tyne, UK

ARTICLE INFO

Article history:
Received 31 July 2015
Accepted 16 January 2016

Keywords:
Inclining experiment
Uncertainty analysis
Ship stability
KG
GM

ABSTRACT

The inclining experiment is typically performed for all new-build ships and after any major refit. The purpose of the inclining experiment is to establish the vertical distance of the centre-of-mass of the ship above its keel in the lightship condition. This value is then taken as the point of reference when loading the ship, for establishing the 'in-service' stability, throughout the life of the ship. Experimental uncertainty analysis is commonly utilised in hydrodynamic testing to establish the uncertainty in a result as a function of the input variables. This can in turn be utilised to establish an interval about the result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurement. This paper provides a methodology for calculating a confidence interval for the location of the centre-of-mass of a ship from an inclining experiment; and ultimately, in any load condition.

The uncertainty compared to an assumed metacentric height of 0.15 m is provided for four classes of ship: buoy tender 0.15 ± 0.15 m ($\pm$100%); super yacht 0.150 ± 0.033 m ($\pm$22.0%); supply ship 0.150 ± 0.047 m ($\pm$31.3%), container ship 0.150 ± 0.029 m ($\pm$19.3%), ropax 0.150 ± 0.077 m ($\pm$100%).

1. Aims and objectives

The aim is to establish procedures for identifying the experimental uncertainty in the estimate of the centre-of-mass height above the keel (referred to as $\overline{KG}$) by method of an inclining experiment (IE).

The first objective is to give procedures for performing a pre-test analysis that can be employed to identify the best course of action for reducing the experimental uncertainty. The second objective is to give procedures for performing a post-test analysis that can be employed to identify a confidence interval for the resulting estimate of $\overline{KG}$.

2. Background

The IE is a required procedure [unless exceptions apply; see IMO, 2008] for all new-build ships and after any major refit. The purpose of the IE is to establish $\overline{KG}$, in the lightship condition. This value is then taken as the point of reference when loading the ship, for establishing the 'in-service' $\overline{KG}$, throughout the life of the ship. An accurate estimate of the limiting $\overline{KG}$ is absolutely necessary for the safe operation of the ship, so as to ensure adequate stability. Clearly, this is dependent on an accurate estimate of the lightship $\overline{KG}$ obtained from the IE.

While typically all attempts are made to conduct the IE in a manner that minimises the introduction of error, many potential sources of error exist. For example, all attempts are made to remove the influence of fluid free-surface effects, by emptying or pressing-full all tanks. Any suspended loads are secured or removed and anything that may move is removed or made secure. Similarly, all attempts are made to conduct the IE in calm conditions, when the effect of wind, waves, current and the wash from passing ships is minimised.

Notwithstanding all attempts to minimise errors, sources of uncertainty will always be present – uncertainty being different from error. Due to the stochastic nature of the world, all input variable measurements are only known with limited accuracy. The uncertainty in the results (in this case the estimate of $\overline{KG}$) is dependent on the magnitude of the uncertainties of each input variable and on the particular sensitivity of the results to each input, which is dependent on the form of the data reduction equations.

* Corresponding author. Tel.: +44 191 222 6750; fax: +44 191 222 5491.
E-mail addresses: michael.woodward@utas.edu.au (M.D. Woodward), m.x.v.rijsbergen@marin.nl (M.v. Rijsbergen), keith.w.hutchinson@babcockinternational.com (K.W. Hutchinson), andrew.scott@mcga.gov.uk (A. Scott).

http://dx.doi.org/10.1016/j.oceaneng.2016.01.017
0029-8018/

2.1. Overview of the inclining experiment

Explanations of the procedure for an IE exist in many texts, with the fundamental description given by (IMO, 2008). In brief, an IE is conducted by forcibly inclining the ship by moving a known weight a known transverse distance across the ship. The inclination is measured from the movement of a plumb-line relative to a mark-board, that is horizontal when the ship is upright. Typically, two or three plumb-lines are employed (forward-amidships-aft) to account for any torsional deformation of the ship. Then, the metacentric height $\overline{GM}$ is obtained according to,

$$\overline{GM}=\frac{wd}{\rho\nabla\tan\theta} \tag{1}$$

where w is the mass of the weight moved, d is the distance the weight is moved, ρ is the water density, ∇ the displaced volume of the ship and θ is the induced heel-angle. Eq. (2) calculates the height of the metacentre above the centre-of-buoyancy as a function-of-form for the given draught.

$$\overline{BM}=\frac{I}{\nabla} \tag{2}$$

In Eq. (2), I is the transverse second moment of area of the water-plane at that draught. The height of the centre-of-buoyancy above the keel $\overline{KB}$, (the centroid of volume at that draught) being a geometric property, is readily calculated from the hydrostatic particulars. The height of the mass-centroid (centre of gravity) above the keel $\overline{KG}$, is then given by Eq. (3).

$$\overline{KG}=\overline{KB}+\overline{BM}-\overline{GM} \tag{3}$$

2.2. Overview of experimental uncertainty analysis

The expression of experimental uncertainty is generally dealt with by National Metrology Institutions. However, for the application of specific procedures, scientific committees or societies more often take responsibility. Considering hydrodynamic testing, the International Towing Tank Conference (ITTC) provides Procedures and Guidelines for many aspects of ship related testing. Though the IE is not within its scope; one procedure (ITTC, 2008) does have relevant information, as it describes the application of uncertainty to hydrodynamic testing. Also, the development of all new procedures and guidelines should be expressed in line with the International Organisation for Standards (ISO), Guide to the Expression of Uncertainty in Measurement (ISO/IEC, 1995).

In accordance with ISO uncertainties can be categorised into Type-A and Type-B. Type-A uncertainties are components obtained utilising a method based on statistical analysis of a series of observations. Type-B uncertainties are components obtained by means other than repeated observations. For the IE most measurements are Type-B; or at least must be treated as such due to the nature of the measurement methods applied. In many respects however, the distinction is arbitrary as, for onward calculations, Type-A and Type-B uncertainties are treated in the same way. In its most simple form, the combined uncertainty in a result $u_c(y)$ is the root-sum-square of the standard uncertainties $u(x_i)$ for each ith input variable multiplied by a corresponding sensitivity coefficient c_i for each variable, given by Eq. (4).

$$u_c^2(y)=\sum_{i=1}^{N}c_i^2u^2(x_i) \tag{4}$$

Of course, this is a somewhat simplified form, neglecting the possibility of correlation between various variables. Such correlation will be dealt with later in the paper, but for the immediate discussion this simplified form is sufficient. The sensitivity coefficient c_i is the partial derivative of the results with respect to any given input variable x_i; given by Eq. (5).

$$c_i=\frac{\partial y}{\partial x_i} \tag{5}$$

The standard uncertainty of any given variable is relatively easy to obtain. If a sufficiently large number of samples of measurement data are available, the Type-A standard uncertainty for a single sample is equal to the sample standard deviation. If there is no recent measurement data available, the limits of the uncertainty need to be estimated or e.g. taken from a specification of a measurement device. With these limits and an assumed probability distribution, the Type-B standard uncertainty can be derived (for application guidance see (ISO/IEC, 1995) Section 4.3).

3. Derivation of sensitivity coefficients

By assuming linearity, for small changes in draught T, for the variables $\overline{KB}$, I and ∇, the sensitivity coefficients can be obtained directly. Going to the hydrostatic tables for the ship, the tangent to the curves at the lightship 'as inclined' draught are utilised to obtain the coefficient α_n and constant terms β_n shown in Eq. (6).

$$\begin{aligned}\overline{KB}&=\alpha_1T+\beta_1\\ I&=\alpha_2T+\beta_2\\ \nabla&=\alpha_3T+\beta_3\end{aligned} \tag{6}$$

Eq. (7) is obtained by substituting Eqs. (1), (2) and (6) back into Eq. (3).

$$KG^{(\alpha_1T+\beta_1)+}\left(\frac{\alpha_2T+\beta_2}{\alpha_3T+\beta_3}\right)-\left[\frac{wd}{\rho(\alpha_3T+\beta_3)\tan\theta}\right] \tag{7}$$

Simplifying as much as possible, the relevant sensitivity coefficients are then given by Eqs. (8)–(12), for the ith heel-angle measurement induced by weight shift. In Eq. (12) the gradient terms α_n are replaced with the specific differential terms, as they are perhaps more meaningful.

$$c_{1i}=\frac{\partial\overline{KG}}{\partial\theta_i}=\frac{wd}{\rho\nabla\sin^2\theta_i} \tag{8}$$

$$c_{2i}=\frac{\partial\overline{KG}}{\partial\rho}=\frac{wd}{\rho^2\nabla\tan\theta_i} \tag{9}$$

$$c_{3i}=\frac{\partial\overline{KG}}{\partial w}=-\frac{d}{\rho\nabla\tan\theta_i} \tag{10}$$

$$c_{4i}=\frac{\partial\overline{KG}}{\partial d}=-\frac{w}{\rho\nabla\tan\theta_i} \tag{11}$$

$$c_{5i}=\frac{\partial\overline{KG}}{\partial T}=\frac{\partial\overline{KB}}{\partial T}+\frac{1}{\nabla}\left(\frac{\partial I}{\partial T}-\frac{\partial\nabla}{\partial T}\overline{BM}+\frac{\partial\nabla}{\partial T}\frac{wd}{\rho\nabla\tan\theta_i}\right) \tag{12}$$

The uncertainty in the ship geometry is an important consideration in comparison to the drawings. This takes into account the uncertainty in the position of the centre-of-buoyance and the metacentre, from which all other calculations are taken. Taking the partial derivatives of Eq. (3) (with Eqs. (1) and (2) substituted accordingly) the sensitivity coefficients given by Eqs. (13)–(15) are obtained.

$$c_6=\frac{\partial\overline{KG}}{\partial\nabla}=\frac{1}{\nabla^2}\left(\frac{wd}{\rho\tan\theta_i}-I\right) \tag{13}$$

$$c_7=\frac{\partial\overline{KG}}{\partial I}=\frac{1}{\nabla} \tag{14}$$

$$c_8 = \frac{\partial \overline{KG}}{\partial \overline{KB}} = 1 \quad (15)$$

4. Identification of the variable uncertainties

With various types of calculation involved in an analysis, a description of uncertainty in 'levels' is more practical. That is to say, use the sensitivity coefficient and standard uncertainty at one level to output the combined uncertainty. Then use this as the input standard uncertainty at the next level. An example of such an approach is implemented within this methodology, utilising the output combined uncertainty for the heel angle measurement as input standard uncertainty for the next calculations. The next section will look at the necessary variables and provides practical methods for obtaining the required values.

4.1. Uncertainty in the heel-angle by plumb-line measurement, $\boldsymbol{u}(\boldsymbol{\theta})$

Taking the length of the plumb-line to be l, and the horizontal measured plumb-line displacement to be η, then the heel angle θ, is given by Eq. (16).

$$\theta = \tan^{-1}\left(\frac{\eta}{l}\right) \quad (16)$$

The combined uncertainty for the measured heel angle is dependent both on the standard uncertainty in l and in η; as given by Eq. (17).

$$u^2(\theta) = \left(\frac{\partial \theta}{\partial l}\right)^2 u^2(l) + \left(\frac{\partial \theta}{\partial \eta}\right)^2 u^2(\eta) \quad (17)$$

Typically, the plumb-line will be swinging back-and-forth in an approximately sinusoidal oscillation. The value for η is typically obtained by trying to estimate the middle of the plumb-line swing. Ideally the estimate of the uncertainty would be obtained as the sample standard deviation of the signal, over a sufficiently large number of cycles. In the case of the IE however, the time history of the plumb-line displacement is typically not recorded. Taking the extremes of the swing would somewhat overestimate the uncertainty. A reasonable estimate for uncertainty in the plumb-line displacement measurement can nevertheless be obtained in terms of the approximate maximum and minimum observed values. The standard deviation of a sinusoidal signal σ_S, of amplitude ζ can be shown to be as given in Eq. (18); with proof provided in Appendix A.

$$\sigma_S = \frac{\zeta}{\sqrt{2}} \quad (18)$$

Assuming that the swinging plumb-line motion is a pure sinusoid, then the signal height is the maximum observed value minus the minimum observed value. The amplitude is by definition half the signal height; given by Eq. (19),

$$\zeta = \frac{\left(s^{max} - s^{min}\right)}{2} \quad (19)$$

where s^{max} is the maximum observed swing of the plumb-line and s^{min} the minimum. Considering that the plumb-line will be oscillating about both the reference position and then later about the measurement position, the uncertainty related to both situations needs to be taken into account. If the magnitude of the oscillations is not far different in either case, the uncertainties in the amplitudes are correlated. Then the standard uncertainty in η is equal to $2\sigma_S$. Substituting Eq. (19) back into Eq. (18), and multiplying by two, the uncertainty in the estimated plumb-line displacement, as giving in Eq. (20), is obtained.

$$u(\eta) = \frac{\left(s^{max} - s^{min}\right)}{\sqrt{2}} \quad (20)$$

If the induced heel-angle is given by Eq. (16), then the sensitivity is the partial derivative of θ with respect to η, given by Eq. (21).

$$\frac{\partial}{\partial \eta}\left[\tan^{-1}\left(\frac{\eta}{l}\right)\right] = \frac{l}{\eta^2 + l^2} \quad (21)$$

In a similar way, the sensitivity with respect to the plumb-line length is given by Eq. (22).

$$\frac{\partial}{\partial l}\left[\tan^{-1}\left(\frac{\eta}{l}\right)\right] = \frac{-\eta}{\eta^2 + l^2} \quad (22)$$

It is important to remember that although several plumb-line measurements are taken at various locations, these are not independent measurements of the same thing. In actual fact, these are discrete measurements each contributing to a part of a data reduction equation. In this case the data reduction equation is rather simplistic, being simply the mean value for N plumb-lines. From this, the sensitivity coefficient for each measurement can be shown to be equal to $\frac{1}{N}$. Bringing together Eqs. (20)–(22) into the form given in Eq. (17), the uncertainty in the heel-angle induced by the ith moment (induced by weight shift) is obtained as given in Eq. (23). Here, the standard uncertainty of the jth plumb-line length $u(l_j)$ is the combination of two uncertainties. The first is the best measurement capability of the measuring equipment utilised to measure it, including components such as calibration uncertainty and resolution. The second is the uncertainty in the measuring process with contributions such as alignment, repeatability.

$$u^2(\theta_i) = \sum_{j=1}^{N}\left(\frac{1}{N}\right)^2 \left\{ \left[\frac{l_j}{\left(\eta_{ji}^2 + l_j^2\right)}\right]^2 \left[\frac{\left(s_{ji}^{max} - s_{ji}^{min}\right)}{\sqrt{2}}\right]^2 + \left[\frac{-\eta_{ji}}{\left(\eta_{ji}^2 + l_j^2\right)}\right]^2 u^2(l_j) \right\} \quad (23)$$

4.2. Uncertainties related to the water density, $\boldsymbol{u}(\boldsymbol{\rho})$

Typically, the water density around the ship will be sampled at several locations and at more than one depth. The average water density is then taken as the basis for subsequent calculations. Utilising this method there are two main areas to be considered. Firstly, there is uncertainty related to the best measurement capability of the device employed to measure the water density. Secondly, there is the uncertainty due to the measuring process.

If for example, the water density is determined by measuring the specific gravity, then the best measurement capability is the combined uncertainty of the calibration uncertainty as provided by the calibration certificate and the resolution (smallest scale division on the gauge), $u(\rho_{bmc})$. The second source of uncertainty to be considered is the uncertainty in the measuring process. The main contribution to this uncertainty is the process of sampling. Since the samples can be assumed to be independent, the standard uncertainty of the mean value can be calculated by dividing the sample standard deviation by the square root of the number of samples, $u(\rho_\sigma)$.

The uncertainty for any necessary temperature correction associated with the hydrometer reading can also be taken into consideration by applying ITTC (2011). However, based on the findings of the case studies (in Section 8), such finesse may be superfluous. The total uncertainty associated with the water density $u(\rho)$, is then given by the root-sum-square of the component uncertainties; given by Eq. (24).

$$u^2(\rho) = u^2(\rho_{bmc}) + u^2(\rho_\sigma) \quad (24)$$

4.3. Uncertainty in the weight of objects moved, $\boldsymbol{u}(\boldsymbol{w_i})$

In an ideal situation, a quayside crane will be employed to move the inclining weights. However, more typically, a forklift truck will be employed to move the inclining weights and then return itself to a known position. Similarly, the staff involved in conducting the IE must also return to known positions before the necessary measurement readings are made. The uncertainty related to items such as the forklift, the personnel and any other equipment are covered in Section 4.7.

The uncertainty of the mass of each inclining weight is assumed to be equal to the calibration uncertainty of the measuring device utilised to weigh it. If a given weight is made up of multiple smaller weights, each having been weighed separately on the same device, then their uncertainties in mass are correlated. This results in a simple addition of the individual uncertainties instead of a root-sum-square calculation. Eq. (25) gives the uncertainty for each ith inclining weight, where N is the number of component weights making up each inclining weight.

$$u(w_i) = \sum_{j=1}^{N} u(w_j) \tag{25}$$

4.4. Uncertainty in the distance objects are moved, $\boldsymbol{u}(\boldsymbol{d})$

When considering the placement of inclining weights, two sources of uncertainty must be taken into account. Specifically, the uncertainty in the location of the marks made for positioning the weights and the uncertainty of the placement of the weights with respect to those marks.

If for example a measurement mark were made on a piece of white paper with a fine pencil and a steel rule calibrated in millimetres, then it would be fair to say that the uncertainty was plus-or-minus a millimetre. Conversely, just because a tape measure calibrated in millimetres is utilised to mark the placement of the inclining weights, to assume such accuracy would be spurious. Stretching a tape-measure across a, perhaps uneven, deck and marking with chalk or sticky-tape, or some such similar crude marking, could be more realistically considered as plus-or-minus a centimetre. Of course, a more sophisticated method might be employed such as a laser measurement, to improve accuracy. Notwithstanding, the task at hand is to make a realistic judgment of the accuracy that can be assumed with the tools utilised. When taking multiple measurements to calculate the total distance the total measurement uncertainty is taken as the root-sum-square of the contributing measurement uncertainties (or simply the sum if the individual measurements are correlated e.g. taken with the same device). Then, the measurement of the mark d_{Mi} relating to the ith inclining weight has an uncertainty $u(d_{Mi})$.

As with the above, when trying to line up an inclining weight (itself on a forklift truck pallet) with a mark made with sticky-tape, then to assume millimetre accuracy would be spurious. As above, the task at hand is to make a realistic judgment of the accuracy that can be assumed with the tools utilised. Then, alignment with respect to the mark d_{Ai} for the ith inclining weight has an uncertainty $u(d_{Ai})$.

For each ith inclining weight moved, the total uncertainty is the root-sum-square of the uncertainty related to the marks and the uncertainty related to the position with respect to the marks. Then, Eq. (26) gives the uncertainty of the distance the ith inclining weight is moved.

$$u^2(d_i) = u^2(d_{Mi}) + u^2(d_{Ai}) \tag{26}$$

4.5. Uncertainties related to the draught marks, $\boldsymbol{u}(\boldsymbol{T})$

The estimate of the draught marks has two sources of uncertainty. The uncertainty related to the position of the draught marks and the uncertainty of the water-level with respect to those marks. For the first of these, the draught mark represents a distance above the keel. The flat bottom of the ship however has itself some variation. Realistically, adjudging the ‘flatness’ of the keel to be, say plus-or-minus 10 mm, then the uncertainty of the draught marks must be at least this. Depending on the construction methods and the quality of build, the task is to make a realistic judgment on the likely building tolerance; here represented by $u(\epsilon_M)$.

In addition to this, the effect of surface tension causes an uncertainty in the exact position of the water level due to the curved meniscus; here represented by $u(\gamma)$. The magnitude of this depends on the roughness of the surface that the fluid is in contact with. A typical value would be in the order of 3 mm and should be added (as a root-sum-square) to the other draught related sources of uncertainty.

As the water surface is invariable moving and, to some extent, the ships itself, then the measurement is problematic. This can be improved upon by the use of a glass tube to damp out the wave action; but some oscillation will always be present. For comparison with the above, typical amplitudes could be in the order of 50 mm. For simplicity, a reasonable estimate of the uncertainty may be obtained by multiplying the oscillation amplitude by the standard deviation of a sinusoidal signal; described in Section 4.1 and Appendix A. Letting the maximum local observed jth draught mark be τ_j^{max} and the minimum be τ_j^{min}, then Eq. (27) gives the combined uncertainty for the draught measurement as,

$$u_c^2(T) = \sum_{j=1}^{3} c_{5j}^2 \left[\left(\frac{\tau_j^{max} - \tau_j^{min}}{2\sqrt{2}} \right)^2 + u^2(\gamma) + u^2(\epsilon_M) \right] \tag{27}$$

where $j=1$ corresponds to the forward draught measurement, 2 the measurement amidships and 3 the aft measurement. Taking into consideration the hog/sag correction and the layer correction, the draught at the longitudinal centre of flotation is given in Eq. (28) (which is typically the reference point in tables describing the ship hydrostatic characteristics),

$$T_{LCF} = \frac{1}{6}(T_1 + 4T_2 + T_3) + LCF\,\frac{(T_3 - T_1)}{L_{bm}} \tag{28}$$

where LCF is the position of the longitudinal centre of flotation with respect to amidships and L_{bm} is the length between draught marks. The corresponding sensitivity coefficients c_{5j} are given by Eqs. (29)–(31).

$$c_{51} = \frac{\partial T_{LCF}}{\partial T_1} = \frac{1}{6} - \frac{LCF}{L_{bm}} \tag{29}$$

$$c_{52} = \frac{\partial T_{LCF}}{\partial T_2} = \frac{4}{6} \tag{30}$$

$$c_{53} = \frac{\partial T_{LCF}}{\partial T_3} = \frac{1}{6} + \frac{LCF}{L_{bm}} \tag{31}$$

By taking an average from N draught measurements and assuming that their uncertainties are independent, the uncertainty of the average draught is given by Eq. (32).

$$u^2(\overline{T}) = \sum_{i=1}^{N} \left(\frac{1}{N}\right)^2 u^2(T_i) \tag{32}$$

4.6. Uncertainties related to hull-form tolerances, $\boldsymbol{u}(\nabla)$, $\boldsymbol{u}(\boldsymbol{I})$ and $\boldsymbol{u}(\overline{\boldsymbol{KB}})$

Taking the usual definition of volume to be $\nabla = LBTC_B$ and taking logarithms, Eq. (33) is obtained.

$$\log \nabla = \log L + \log B + \log T + \log C_B \tag{33}$$

Recognising that if $y = \log x$ then $\frac{dy}{dx = \frac{1}{x}}$ so $dy = \frac{dx}{x}$, Eq. (34) is obtained.

$$\frac{\partial \nabla}{\nabla} = \frac{\partial L}{L} + \frac{\partial B}{B} + \frac{\partial T}{T} + \frac{\partial C_B}{C_B} \tag{34}$$

Considering the change in any given parameter to be the manufacturing tolerance in that given dimension (denoted ϵ), then Eq. (34) can be rewritten. To assign a tolerance to the block coefficient an assumption is made that any horizontal transverse measurement from the centre-line has the same tolerance as that of the breadth. This leads to a simplification (factor of 2 on breadth tolerance) where Eq. (35) gives the uncertainty in displaced volume.

$$u(\nabla) = \nabla\left(\frac{\epsilon_L}{L} + 2\frac{\epsilon_B}{B} + \frac{\epsilon_T}{T}\right) \tag{35}$$

In a similar way, assuming that the water-plane area can be approximated by a rectangle, the second moment of area is given by $I = \frac{LB^3}{12}$. Again taking logarithms and with the same process as above, Eq. (36) gives the uncertainty in the transverse second moment of water-plane area.

$$u(I) = I\left(\frac{\epsilon_L}{L} + 3\frac{\epsilon_B}{B}\right) \tag{36}$$

From a similar analogy, Eq. (37) gives the uncertainty in the height of the centre of buoyancy.

$$u(\overline{KB}) = \overline{KB}\left(\frac{\epsilon_T}{T}\right) \tag{37}$$

4.7. Uncertainties related to the removal or addition of weights $\boldsymbol{u}(\boldsymbol{\delta G})$

It is necessary to remove the inclining weights and other equipment from the ship after the IE is finished. The estimate of $\overline{KG}$ must then be amended accordingly. In addition, though not ideal, the ship may well have weights on-board that will be removed or still to be added. Eq. (38) gives a change in the position of the ships centre-of-gravity due to the addition on removal of an ith weight of vertical distance h_i from the original centre-of-gravity (w_i will be a negative value for the removal of a weight).

$$\delta G_i = \frac{h_i w_i}{\Delta + w_i} \tag{38}$$

The sensitivity coefficients for a shift in the centre-of-gravity, due to the addition or removal of an ith weight are given in Eqs. (39)–(41).

$$c_{9i} = \frac{\partial(\delta G_i)}{\partial w_i} = \frac{h_i \Delta}{(\Delta + w_i)^2} \tag{39}$$

$$c_{10i} = \frac{\partial(\delta G_i)}{\partial h_i} = \frac{w_i}{\Delta + w_i} \tag{40}$$

$$c_{11i} = \frac{\partial(\delta G_i)}{\partial \Delta} = \frac{-w_i h_i}{(\Delta + w_i)^2} \tag{41}$$

The standard uncertainty of the mass of the ith weight $u(w_i)$ and the height of the ith weight $u(h_i)$ should be taken as the combined uncertainty of the calibration uncertainty of the devices utilised to measure them (or a realistic estimate) and the uncertainty in the measurement. The standard uncertainty for the displacement $u(\Delta)$ can be obtained from the density and volume uncertainties (given in Eqs. (24) and (35) respectively) by Eq. (42).

$$u(\Delta) = \Delta\left[\frac{u(\nabla)}{\nabla} + \frac{u(\rho)}{\rho}\right] \tag{42}$$

4.8. Uncertainties related to free-surface corrections $\boldsymbol{u}(\boldsymbol{FSC})$

After the IE is conducted a correction to $\overline{KG}$ may be required-if there are any free-surfaces aboard the ship during the test. Assuming tanks to be approximately rectangular, the free-surface correction is given by Eq. (43). In the equation ϱ_i is the density of the fluid in the ith tank and a_i and b_i are the length and breadth of that tank respectively.

$$FSC = \frac{\varrho_i}{\rho}\frac{a_i b_i^3}{12\nabla} \tag{43}$$

The sensitivity coefficients for the free-surface correction are given in Eqs. (44)–(48).

$$c_{12i} = \frac{\partial FSC}{\partial \varrho_i} = \frac{1}{\rho}\frac{a_i b_i^3}{12\nabla} \tag{44}$$

$$c_{13i} = \frac{\partial FSC}{\partial \rho} = -\frac{\varrho_i}{\rho^2}\frac{a_i b_i^3}{12\nabla} \tag{45}$$

$$c_{14i} = \frac{\partial FSC}{\partial a} = \frac{\varrho_i}{\rho}\frac{b_i^3}{12\nabla} \tag{46}$$

$$c_{15i} = \frac{\partial FSC}{\partial b} = \frac{\varrho_i}{\rho}\frac{a_i b_i^2}{4\nabla} \tag{47}$$

$$c_{16i} = \frac{\partial FSC}{\partial \nabla} = -\frac{\varrho_i}{\rho}\frac{a_i b_i^2}{12\nabla^2} \tag{48}$$

The standard uncertainty for the density of fluid in the ith tank $u(\varrho_i)$ is obtained in a similar way as the uncertainty for the seawater density $u(\rho)$; see Section 4.2. The standard uncertainties for the length a_i and breadth b_i of each tank are taken as the calibration uncertainty of the device utilised to measure them, and the uncertainty in ship displaced volume $u(\nabla)$ as given in Eq. (34).

4.9. Other sources of uncertainty

4.9.1. Uncertainty of the position of the inclining weight centroid

While methods do exist for finding the centroid of a mass by direct measurement, they are unlikely to be undertaken. Provided the inclining weights are not rotated when moved, the position of the centroid is not important. That is to say, the distance moved by the centroid will be the same as the distance moved by any point of reference. Therefore, careful attention to the procedure can remove this source of uncertainty.

4.9.2. Uncertainty of the marks made on deck for longitudinal placement

The difficulty with the longitudinal marks is more one of finding a suitable point of reference. If a hatch combing or accommodation block bullhead is utilised for reference, then the uncertainty in their placement must be considered. Sighting transversely across the deck, at right angles to the parallel-mid-body, is again not without difficulties. Considering this, an uncertainty of approximately 10 cm is reasonable. While this may sound alarmingly large, remember this value will be multiplied by a sensitivity coefficient. This then considers the sensitivity coefficient for the change in I and ∇ with respect to a small change in trim. These terms will be negligibly small provided the trim is minimal.

4.9.3. Uncertainties when utilising ballast tanks as inclining weights

In some cases the general arrangements of the ship prohibit the use of mobile inclining weights. In such cases, the ballast tanks are employed as an alternative. For example, a port side tank may be filled. Then, when ready, the tank will be emptied and an equivalent tank on the opposite side filled. In such cases, the uncertainty is related to the relative positions of the centroid of each tank, the volume of each tank and the density of the fluid used to fill them. Taking the root-sum-square for these items then the sensitivity can be taken with respect to the induced moment. Also, the uncertainty in any free-surface correction must be taken into account.

5. Combined uncertainty

It is not uncommon in an IE to take multiple measurements by additional or repeated weight movements. Estimates of the random uncertainty from the standard deviation of the mean are possible, if multiple truly independent measurements are made. This however provides only the uncertainty in the estimate of $\overline{GM}$ and not $\overline{KG}$. The estimates of $\overline{KB}$ and $\overline{BM}$, both necessary for the estimate of $\overline{KG}$, are dependent on parameters also measured as part of the IE; and must be dealt with appropriately. Notwithstanding, more likely the individual measurements are not truly independent. For example, the second induced angle may include the moment from both the first and second inclining weights. Similarly, a third weight move may be achieved by returning the first weight to its original position. The uncertainties should thus be assumed to be fully correlated and combined accordingly. As, in this case, the data reduction equation is a simple average then the uncertainty for fully correlated variables is also a simple average, given by Eq. (49).

$$u_c\left(\overline{KG}\right) = \frac{1}{N}\sum_{i=1}^{N} u\left(\overline{KG}_i\right) \tag{49}$$

For items that are to be removed (as described in Sections 4.7 and 4.8), the corresponding uncertainties should be included after the samples of $u\left(\overline{KG}_i\right)$ are combined utilising Eq. (49).

6. Expanded uncertainty (*U*)

The combined uncertainty is defined as equivalent to one standard deviation. This corresponds to a confidence interval of approximately 68% if the uncertainty can be assumed to be normally distributed. In engineering applications a higher confidence interval when expressing the uncertainty is more practical. This can simply be achieved by multiplying the combined uncertainty u_c by a coverage factor k, which gives the expanded uncertainty U. For example, assuming a normal distribution, $k=2$ gives a U_{95} with a 95% confidence interval.

7. Method

An experimental uncertainty analysis may be performed prior to the IE, as a process of experimental design, or post analysis to establish a confidence interval in the result. The main difference is that, prior to the test being conducted, the limits of some parameters must be estimated. In either case the calculations are relatively straightforward and can be performed easily with a typical spread-sheet application. Also in either case, the process is predominantly the same and can be structured into seven key steps as described in Fig. 1.

Utilising the methods outlined for Step 6 (Sections 4.7 and 4.8), the uncertainty in $\overline{GM}$ for any load condition can readily be obtained.

8. Case studies

To establish the fitness-for-purposes of the procedure and to meet with the objectives of the paper, the procedure is applied to five case-study ship inclining experiments. In line with the objectives of the paper, the results are utilised to find the uncertainty in the estimated $\overline{KG}$ and, explore the origins of contributing uncertainties to help target improvements in the experimental procedures.

As the data is historic, not all of the necessary parameters specified by this procedure are available. Nevertheless, the data serves perfectly well to perform a typical pre-test analysis. This has in fact some advantages in that environmental inputs are made the same for all five ships, making the results more directly

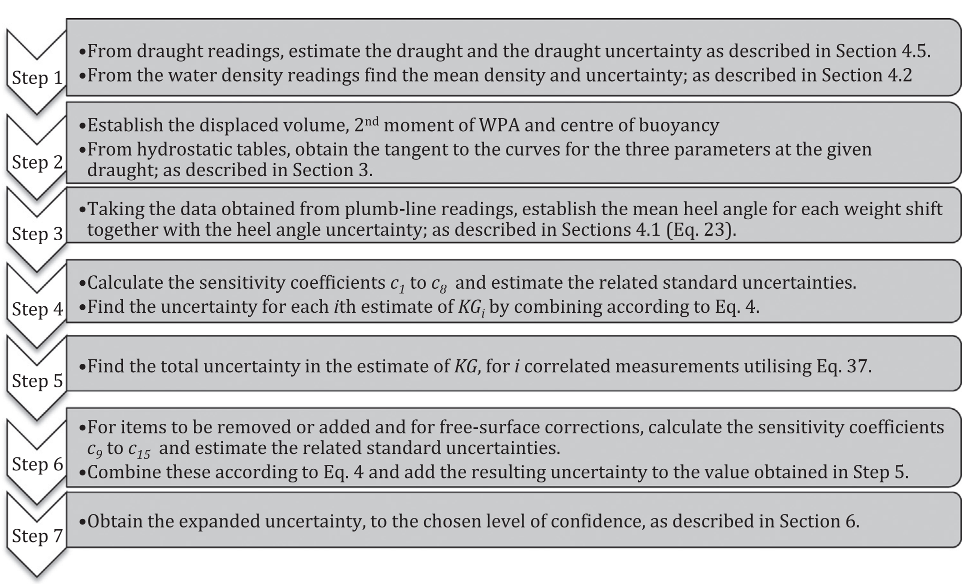

Fig. 1. Step-by-step flow chart for the implementation of the uncertainty analysis procedure for an inclining experiment, either pre- or post-analysis.

Table 1
Results from case study ships.

Parameter (units)	L_{bp} (m)	Δ_{Design} (tonne)	$\overline{KG}$ (m)	$u_c(\overline{KG})$ (m)	U_{95} (m)	$U_{95}(\overline{GM})$ (%)[a]
Buoy tender	37	453	3.580	0.075	0.15	100
Super yacht	50	698	4.340	0.016	0.033	22.0
Supply ship	51	904	4.173	0.024	0.047	31.3
Container	124	15,718	10.245	0.014	0.029	19.3
Ropax	204	23,370	16.620	0.077	0.15	100

[a] The expanded uncertainty is given as a % of an assumed metacentric height of 0.15 m.

comparable. For example, in all cases wave amplitudes of 5 cm are assumed for the water surface when taking draught readings. Similarly, plumb-line readings are all assumed to be oscillating with amplitude of 1mm and the water density is assumed to have a 5 kg/m^3 standard uncertainty in all cases. In all cases the manufacturing tolerances are assumed to be ± 10 mm in the length and ± 3 mm in all other dimensions.

For commercial considerations, the full details of the particular ships are not published. Table 1 contains however all of the pertinent values necessary to form a judgment. For reference, the table gives the length between perpendiculars and the design displacement for each ship, together with a descriptive ship-type title. In each case, the estimated $\overline{KG}$ is given together with the combined uncertainty and the expanded uncertainty for a 95% confidence interval. This contains only the uncertainty associated with the light ship estimate and not the uncertainty of all other items (cargo, fuel, water, ballast etc.) on-board the ship in its loaded condition. The uncertainty of the light-ship $\overline{KG}$ is the minimum possible values and the implication of this for the operation of the ship is certainly worth considering. Comparing the uncertainty in $\overline{KG}$ to the value of $\overline{KG}$ is not particularly meaningful in this case as the magnitude of $\overline{KG}$ is somewhat arbitrary, and will change as the ship is loaded. Strictly speaking, the uncertainty in the position of the centre of mass (G) is important and not its magnitude with respect to an arbitrary reference point such as the keel (K). Consideration of the magnitude of the expanded uncertainty for a typical $\overline{GM}$ limitation is perhaps more meaningful. Considering the basic IMO requirement for $\overline{GM}$ to be greater than 0.15 m, the given values of expanded uncertainty can simply be added on to find the necessary $\overline{GM}$ that would have a 95% confidence of achieving the given criterion. For comparison, the percentage of expanded uncertainty with respect to an assumed $\overline{GM}$ of 0.15 m is given in the last column of Table 1.

For the three smaller ships, if the confidence interval encompasses a potentially negative $\overline{GM}$, this does not necessarily present a problem, as they would not normally be loaded to this limit (or be required to do so). In the case of the container ship however $\overline{GM}$ would typically be close to this limit; to prevent high roll accelerations that might otherwise cause damage to the container stacks. In this case the ship would have to be loaded to a $\overline{GM}$ value of nearly 0.18 m to ensure a 95% confidence of compliance. Similarly, the Ropax would typically load close to the limiting $\overline{GM}$ to reduce the risk of high acceleration causing a shift of cargo. In this case the ship would have to be loaded to a $\overline{GM}$ value of nearly 0.33 m; more than double the criterion limit. Note, this estimate does not accounting for uncertainties in the loading of the ship; that could be much larger.

It is clear from the results that the magnitude of estimated uncertainty varies widely for the ships considered; with at least one, the Buoy tender, showing a markedly high value. To better explore the origins of the uncertainties, the contributions from various inputs are examined. Fig. 2 gives the uncertainties for

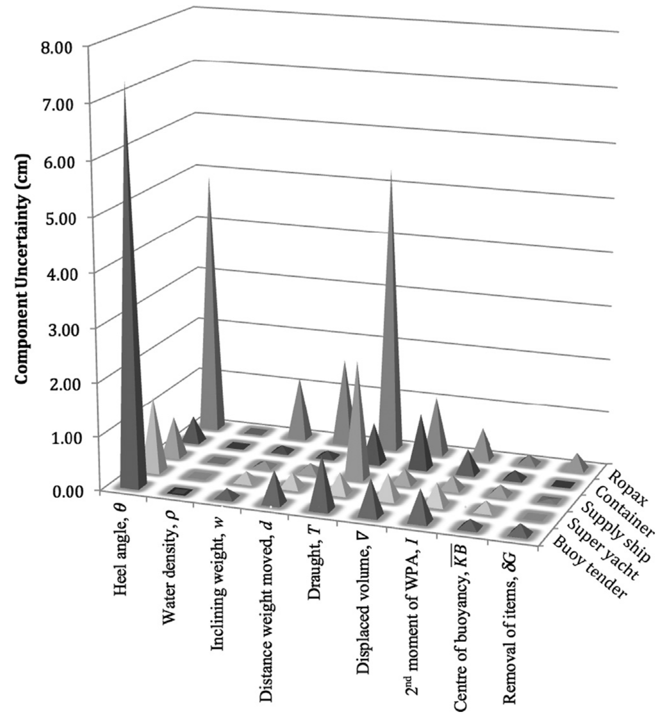

Fig. 2. Component uncertainty contribution in the vertical location of the centre of mass for various inclining experiment parameters for the five case study ships.

various inputs for each ship. On examination the importance of heel angle is clear; and notably so for both the Buoy tender and the Ropax. The Buoy tender does have the smallest average induced heel angle ($\approx 0.6°$). The Ropax however has an average induced heel angle in line with and sometimes greater that the other ships examined. This is a function of the sensitivity of the results to the heel angle that depends on the relationship between various parameters (heel angle; plumb-line length; plumb-line oscillation; $\overline{GM}$). This perhaps exemplifies well the value of performing a pre-test uncertainty analysis, to avoid such situations. Notwithstanding, the two parameters here that may be readily controlled are the induce heel angle (which should be appropriately large) and the plumb-line length (which should be as long as possible).

The second most influential parameter appears to be the draught measurement. In actual fact, the 5 cm wave amplitude is most likely very optimistic, and could be much larger. Nevertheless, repeating the draught measurement more than once quickly reduces the uncertainty in this parameter. Establishing the minimum number of necessary draught measurements needed for any particular wave condition is a relatively easy process using this procedure.

The next most important parameters appear to be the estimate of displaced volume, followed by the estimate of the second moment of water plane area. These are dependent on the manufacturing tolerances, and the estimate thereof. Of course, this can vary depending on the shipyard. More sophisticated ways of measuring the ‘as-built’ form/dimensions may be considered if this parameter is identified as significant.

It is worth also considering the inferred relationships from the sensitivity coefficients. Assuming that the ship is simply a box with the same length, breadth and draught but with a block coefficient tending to unity, then Eq. (12) can be substantially simplified. The centre of buoyancy of a box is always at half the draught, so $\frac{\partial \overline{KB}}{\partial T} = 0.5$. Also, the second moment of water plane area does not change with draught, so $\frac{\partial I}{\partial T} = 0$. Substituting also Eq. (1) and recognising that $\overline{GM} - \overline{BM} = \overline{BG}$, Eq. (12) can be reduced to the simplified form given in Eq. (50).

$$\frac{\partial \overline{KG}}{\partial T} = 0.5 - \frac{\partial \nabla}{\partial T}\frac{\overline{BG}}{\nabla} \tag{50}$$

This indicates that, to reduce sensitivity, $\overline{BG}$ must be as high as possible. As the height of the centre-of-buoyancy at a particular draught is fixed by the geometry of the ship, a more generally inference can be made in that the centre of gravity must be as high as possible. Also Eq. (50) indicates that ∇ must be as small as possible. Inspection of Eq. (1) shows that both situation result in increased induced heel angles. Some caution should be exercised however as, while large heel angles may reduce uncertainty, they will at the same time increase error due to changes in the position of the metacentre. Nevertheless, heel angles in excess of 7° would be needed before metacentric theory is seriously compromised; far in excess of those needed for a successful IE.

9. Conclusions and recommendations

The aim of this study was to establish procedures for identifying the experimental uncertainty in the estimate of $\overline{KG}$, obtained by IE. The objective were to give procedures for performing a pre-test analysis to help reducing the experimental uncertainty and post-test analysis to identify a confidence interval for the resulting estimate of $\overline{KG}$.

A procedure is provided together with case studies, demonstrating how the uncertainty in an IE can be utilised to improve the design of the experiment. No one parameter can be identified in all cases as problematic from the case studies. There is however a strong indication that the uncertainty in the heel-angle measurement is important but this may be a function of other factors such as $\overline{GM}$. Nevertheless, the longest possible plumb-line (or perhaps an electronic alternative) with sufficiently large induce heel angles should help to reduce uncertainty. The draught measurement uncertainty was also seen to be important, but can be substantially improved with increased sample size. Also, the knowledge of the ‘as-built’ condition in terms of manufacturing tolerances was identified as important. If this were identified as critical for any particular ship, alternative methods could be employed to establish the as build dimensions more accurately.

A procedure is provided for estimating a confidence interval for $\overline{KG}$ and argued to be more usefully considered as a confidence interval for $\overline{GM}$. The case studies show that, for some ships, a substantial increase in the minimum $\overline{GM}$ may be necessary to ensure safe operation.

In addition to the original objectives, the methods outlined for the addition or removal of weights and for free-surface correction, provide a full and complete procedure for establishing the uncertainty in $\overline{GM}$ for any load condition.

Appendix A. Standard deviation of a sinusoidal signal

Taking the definition of standard deviation to be given by Eq. (A1.1), where x_i is the ith sample amplitude, μ is the mean value of all samples and N is the number of samples.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} \tag{A1.1}$$

The mean value μ, of a sinusoidal signal, between the limits of zero and $\frac{2\pi}{\omega}$, will be by definition zero. Then, replacing x_i with $\zeta \sin \omega t$ [where ζ is the amplitude, ω is the frequency and t is time] we get Eq. (A1.2).

$$\sigma_S = \sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \zeta^2 \sin^2 \omega t dt} \tag{A1.2}$$

Integrating between the given limits, gives:

$$\zeta^2 \int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t dt = \zeta^2 \left[\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega}\right]_0^{\frac{2\pi}{\omega}}$$

which, by substituting in the values for the limits, can be seen to equal $\frac{\zeta^2 \pi}{\omega}$. Substituting this back into Eq. (A1.2), we obtain:

$$\sigma_S = \sqrt{\frac{\omega}{2\pi}\frac{\zeta^2 \pi}{\omega}}$$

Cancelling out, the standard deviation of a sinusoidal signal for any number of whole cycles, is by definition thus given by Eq. (A1.3).

$$\sigma_S = \frac{\zeta}{\sqrt{2}} \tag{A1.3}$$

References

IMO, International Code on Intact Stability, 2008 (2009 Edition), IMO Publishing, ISBN: 978-92-801-1501-7, MSC 85/26/Add.1 ANNEX 2, pp. 71–89.

ISO/IEC Guide 98-3:2008, Uncertainty of Measurement – Part 3: Guide to the Expression of Uncertainty in Measurement (GUM:1995).

ITTC, Fresh Water and Seawater Properties, ITTC Procedure 7.5-02-01-03, Effective Date 2011, Revision 02.

ITTC, Recommended Procedures and Guidelines: Guide to the Expression of Uncertainty in Experimental Hydrodynamics, 7.5-02-01-01, Effective Date 2008, Revision 01, pp. 27.