ABSTRACT

This paper proposes a practical fuel budget problem which aims to determine a group of bunker fuel budget values for a liner container ship over a round voyage under uncertainties caused by severe weather conditions. The proposed problem holds a kernel position in the ship fuel efficiency management programs advocated by container shipping lines due to the downward pressure of soaring bunker prices, according to our research collaboration with a global container shipping line in Singapore. We consider the synergetic influence of sailing speed and weather conditions on ship fuel consumption rate when estimating the bunker fuel budget of a ship over a round voyage. To address the adverse random perturbation of fuel consumption rate under severe weather conditions, we employ the state-of-the-art robust optimization techniques and develop a robust optimization model for the fuel budget problem. The developed model can be dualized into a mixed-integer linear programming model which may be solved by commercial optimization solvers. However, algorithmic findings in the field of robust optimization provide a polynomial time solution algorithm, and we retrofit it to accommodate the proposed ship fuel budget problem. The case study on an Asia-Europe service demonstrates the computational performance of the proposed solution algorithm, and the competence of the proposed robust optimization model to produce fuel budget values at different levels of conservatism possessed by the fuel efficiency specialists in container shipping lines.

KEY WORDS:

Fuel consumption, budget, sailing speed, weather condition, robust optimization

INTRODUCTION

Bunker fuel prices have been soaring in the past years from about 200 USD/MT to around 600 USD/MT. For instance, the spot market price of IFO 380 in Singapore increased from lower than 300 USD/MT in the first quarter of 2009 to higher than 700 USD/MT at the same period of 2012, and has remained above 600 USD/MT since then. High bunker prices make bunker cost become a large portion of the operating costs for a container shipping line. Ronen (*1*) points out that bunker cost will account for three quarters of the total operating costs of a large container ship if the bunker fuel price exceeds 500 USD/MT. This poses considerable downward pressure on the revenue of container shipping lines. To make things worse, the current economic crisis has resulted in the slump of shipping demand which further crushes the profit margins of container shipping lines.

To relieve the financial burden caused by the increasing bunker cost, container shipping lines have been advocating ship fuel efficiency management programs of various kinds. In a ship fuel efficiency management program, budgeting the fuel consumption of each container ship in the fleet over a planning horizon (say over a round voyage) is of significant importance. In fact, the bunker fuel budget problem for each container ship forms the basis of the entire ship fuel efficiency management program. In the strategic or tactical level, to allocate bunker budget among various shipping routes, one needs to estimate the fuel consumption of each container ship over each round voyage. In the operational level, fuel efficiency specialists in a container shipping line have to clearly understand the fuel consumption profile of each container ship over a round voyage at different operational conditions to provide benchmarks for implementing an ask-and-inspection fuel control mechanism between captains on board and on-shore officers.

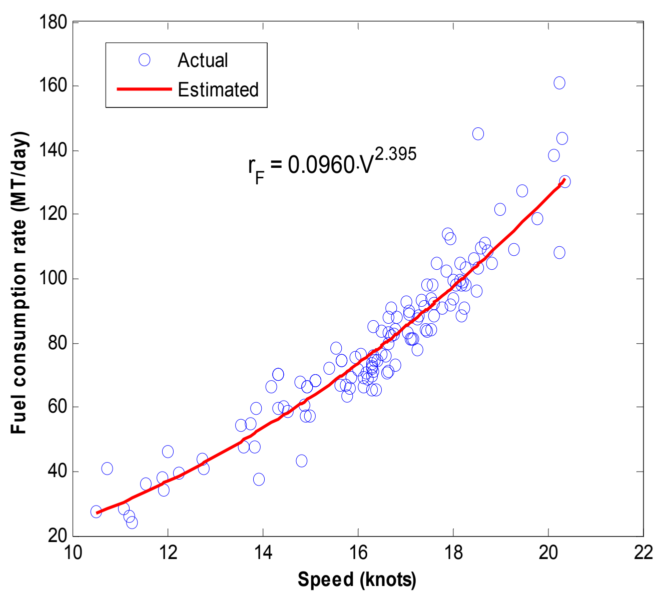

FIGURE1 Fuel consumption rate of a 13000-TEU container ship (S1) at different speeds: R^2 = 0.9080

However, it is challenging to precisely estimate the bunker fuel consumption of a container ship in a planning horizon, even over a round voyage, since the fuel consumption of a ship in a time unit (say one day) is influenced by many factors, such as its sailing speed, displacement, trim, and weather/sea conditions experienced, in an extremely complicated way (*2*). Among these factors, sailing speed is the main determinant. Figure 1 illustrates a quantitative relationship between the fuel consumption rate (r_F) of a 13000-TEU container ship (labeled as "Ship S1" hereinafter) and its sailing speed (V), based on real data collected from a global container shipping line. It can be seen that the sailing speed can explain up to 90% of the fuel consumption. However, it should be noted that weather conditions will also significantly affect the fuel consumption rate. Figure 2 depicts the fuel consumption rate of ship S1 in bow waves at different sailing speeds. We can observe that the fuel consumption of ship S1 in one day increases dramatically with wave heights when the ship experiences bow waves. In reality, the influence of sailing speed and that of weather conditions (wind, waves) are coupled together in a sophisticated way (*3*).

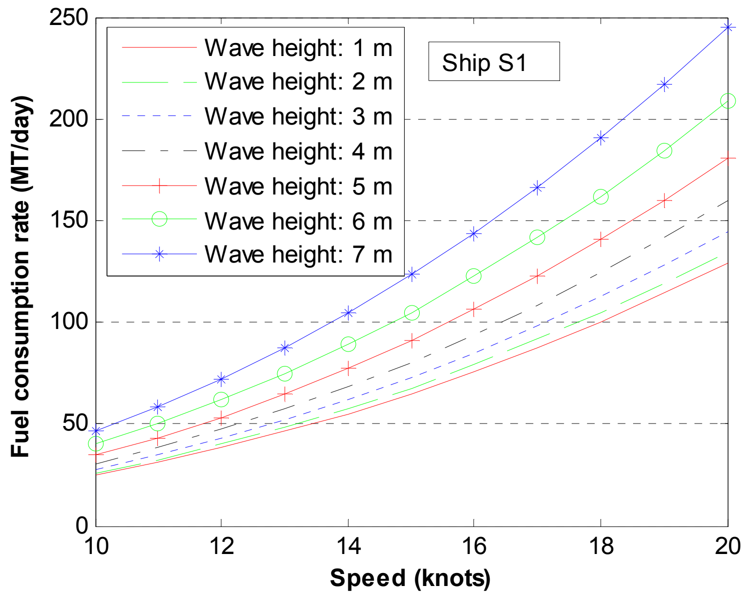

FIGURE 2 Fuel consumption rate of a 13000-TEU container ship (S1) in bow waves

The influence of sailing speed on fuel consumption rate has recently been well recognized by the maritime studies because it plays an important role in liner shipping network analysis (*4-6*), including shipping network design (*7*), ship fleet deployment (*8*), ship schedule design (*9*), container assignment (*10*), and cargo booking and routing (*11*). Notteboom and Vernimmen (*12*), and Ronen (*1*) analyze the relationship between bunker price, sailing speed, service frequency and the number of ships on a shipping route. Álvarez (*8*) takes the ships of different speeds as different ship types when examining the joint routing and deployment of a fleet of container ships, and quantifies the bunker cost in the objective of his model. Fagerholt et al. (*13*) discretize the arrival time (equivalently the sailing speed) at each port call and formulate the ship speed optimization over a single voyage as a shortest path problem. Brouer et al. (*7*) implicitly consider the sailing speed optimization in liner shipping network design by experimentally evaluating several possible vessel-speed-route combinations and selecting the most promising one. Wang and Meng (*14*), and Qi and Song (*9*) minimize the bunker fuel consumption of ships by speed optimization under operational uncertainties, such as random port durations and sea contingency. Golias et al. (*15*) and Du et al. (*16*) study the berth allocation problem considering fuel consumption to evaluate the performance of the virtual arrival policy.

Although the influence of weather conditions on ship fuel consumption rate was revealed several decades ago from the viewpoint of naval architecture (*17; 18*), it is seldom considered by existing liner shipping studies. The weather routing problem (WRP) of ships exhibits the impact of weather on ship transit time and sea-keeping (*19-21*). Unfortunately, it overlooks the influence of weather conditions on ship fuel consumption. Lin et al. (*22*) capture the influence of weather conditions on fuel consumption during sailing in their three-dimensional modified isochrones method. However, the propeller resolution speed of the ship along the optimal route is assumed to be constant.

We note that the synergetic influence of sailing speed and weather conditions on fuel consumption of ships is usually ignored. The uncertainties in ship fuel consumption rates caused by variable weather conditions are not captured by existing studies. Furthermore, more importantly, studies on budgeting ship fuel consumption in a planning horizon, which intrinsically requires to consider the synergetic influence of sailing speed and weather conditions and the uncertainties in fuel consumption resulting from variable weather conditions, are not found. This poses a gap between industrial needs and academic studies.

Objectives and Contributions

This study deals with the fuel consumption budget problem of a single container ship over a round voyage by incorporating the coupled influence of sailing speed and weather conditions and the uncertainties in fuel consumption, utilizing the state-of-the-art robust optimization techniques (*23-25*). The robust optimization model and the corresponding solution algorithm, which will be presented in the subsequent sections, can produce different fuel budget values reflecting different conservatism levels of fuel efficiency specialists in container shipping lines.

The contributions of this study are threefold: (a) this study proposes the fuel consumption budget problem of a single container ship over a round voyage, which is a new research topic in maritime studies; (b) it addresses the synergetic influence of sailing speed and weather conditions on ship fuel consumption which is seldom considered in literature; and (c) this study takes an initiative to extend the applications of robust optimization approaches to liner shipping network analysis.

The remainder of this paper is organized as follows. We first introduce the fuel consumption budget problem for a single container ship over a round voyage, and build a nominal mathematical model. Then, we proceed to develop a robust optimization model to address the fuel consumption uncertainty over each sailing leg. Third, we give a polynomial time algorithm according to the theoretical findings of Bertsimas and Sim (*24*) on robust optimization. At last, we report experimental results and conclude this study.

FUEL CONSUMPTION BUDGET PROBLEM FOR A SINGLE CONTAINER SHIP AND THE NOMINAL MATHEMATICAL MODEL

Problem Statement

Consider a liner shipping service operated by a container shipping line. A round voyage of a liner shipping service typically consists of a sequence of port calls: $1 \to 2 \to 3 ... \to k \to k+1 \to ... \to N \to N+1$, in which the $(N+1)^{th}$, namely the last, port call represents the same container port as the first call. The voyage from the k^{th} to $(k+1)^{th}$ port call is referred to as the sailing *leg* k of the service, $k \in \{1, 2, ..., N\}$. For each port call k, each ship deployed should comply with an arrival time window $\left[a_k^{EARLY}, a_k^{LATE}\right]$ and stay at this port with time duration p_k (hours). Meanwhile, denote the sailing distance of leg k by d_k (n mile). Take the LP4 service operated by American President Lines (APL) in Table 1 for example, there are totally 14 port calls: Ningbo (NTB) is the first port call, and the subsequent Yangshan (YAN), Yantian (YAT) and Singapore (SIN) are the 2nd, 3rd and 4th port call. Among 13 sailing legs, Hamburg (HF8) to Rotterdam (RTM) is the 8th one which is 225-nm long. If we defined the departure time from Ningbo as time zero, the ship should arrive at Rotterdam after sailing over leg 8 between time 888 and 912 (hours). After experiencing 45 hours of maneuvering, anchoring, piloting and container handling, the ship will leave Rotterdam and begins its long-time sailing over leg 9 to the Suez Cannel (SUZ) which is virtually considered as a port.

TABLE 1 Shipping schedule of service LP4 published by APL

Sailing leg			Port Time window (destination port)		
Origin	Destination	Distance	Duration	Early arrival [a]	Late arrival
NTB	YAN	80	40	0	24
YAN	YAT	700	16	72	96
YAT	SIN	1430	31	192	216
SIN	SUZ	5020	18	528	552
SUZ	KLV	3130	19	720	744
KLV	SOU	70	35	744	768
SOU	HF8	425	50	816	840
HF8	RTM	225	45	888	912
RTM	SUZ	3350	22	1176	1200
SUZ	JED	625	31	1248	1272
JED	SIN	4420	58	1560	1584
SIN	YAT	1450	20	1728	1752
YAT	NTB	705	32	1800	1816

Note: [a] when an arrival time window is discretized on an hourly basis, the earliest arrival time is "Early arrival" plus 1.

If we construct a shipping schedule with the arrival time at port call $k \in \{1,2,...,N,N+1\}$ being a_k and define the departure time from the first port call $t_1^{DEPART} = a_1 + p_1 = 0$ (so that the N^{th} sailing leg can be treated in the same way as other legs), then the transit time t_k of the ship over sailing leg k should be $a_{k+1} - (a_k + p_k)$ hours, and sailing speed v_k should be maintained at $d_k / (a_{k+1} - (a_k + p_k))$ knots. Given the following power function relationship between fuel consumption rate (r_F, MT/h) and its sailing speed V:

$$r_F = c_1 \cdot V^{c_2} \tag{1}$$

as illustrated in Figure 1, the total bunker fuel consumption of this ship over the whole round voyage can be calculated as

$$\begin{aligned} F &= \sum_{k=1}^{N} c_1 (v_k)^{c_2} \cdot t_k = \sum_{k=1}^{N} c_1 \left(\frac{d_k}{a_{k+1} - (a_k + p_k)} \right)^{c_2} \cdot (a_{k+1} - (a_k + p_k)) \\ &= \sum_{k=1}^{N} c_1 (d_k)^{c_2} \cdot (a_{k+1} - (a_k + p_k))^{1-c_2} \end{aligned} \tag{2}$$

It can be seen that the sailing schedule $\{a_k\}_{k=1}^{N+1}$ determines the total fuel consumption of the ship over a round voyage.

The bunker fuel budge problem in this study attempts to find an optimal sailing schedule to minimize the total fuel consumption of a ship over a whole round voyage. Meanwhile, due to the adverse influence of weather conditions, the fuel consumption rate of the ship over each leg k might change (consider only increase here for our purpose of budgeting fuel consumption with upper limits) randomly, but within a pre-definable interval $\left[c_1\left(v_k\right)^{c_2}, c_1\left(v_k\right)^{c_2}+\delta_k\right]$, where δ_k can be obtained by the historical weather records and the regression results similar to those in Figure 2. Our objective is to construct robust sailing schedules to minimize the total fuel consumption of a ship over a round voyage under the uncertainties in ship fuel consumption rates caused by weather conditions, which would provide credible fuel consumption budget values of a ship over a round voyage in a more realistic sense and thus some useful benchmark values for ship fuel efficiency specialists in shipping lines.

Nominal Mathematical Model

If we do not consider the perturbation (uncertainties) of the fuel consumption rates during sailing, the bunker fuel budget problem can be easily formulated and solved below by following the elegant approach proposed by Fagerholt et al. (*13*). We first discretize the arrival time window $\left[a_k^{EARLY}, a_k^{LATE}\right]$ at k^{th} port call into N_k values, denoted by $A_k=\left\{a_k^i\right\}_{i=1}^{N_k}$, and determining an arrival time at this port call is nothing but to chose a value in A_k. With this discretization of arrival time windows, the nominal fuel budget problem for a ship over a round voyage boils down to a shortest path problem shown in Figure 3. Let f_k^{ij} be the fuel consumption rate of the ship over the link from the node representing a_k^i to that for a_{k+1}^j, then the cost, namely the fuel consumption, over this link is $f_k^{ij}\cdot\left(a_{k+1}^j-\left(a_k^i+p_k\right)\right)$. Finding a minimal fuel consumption schedule is to find a shortest path from the first node to one of the nodes in the $\left(N+1\right)^{th}$ layer.

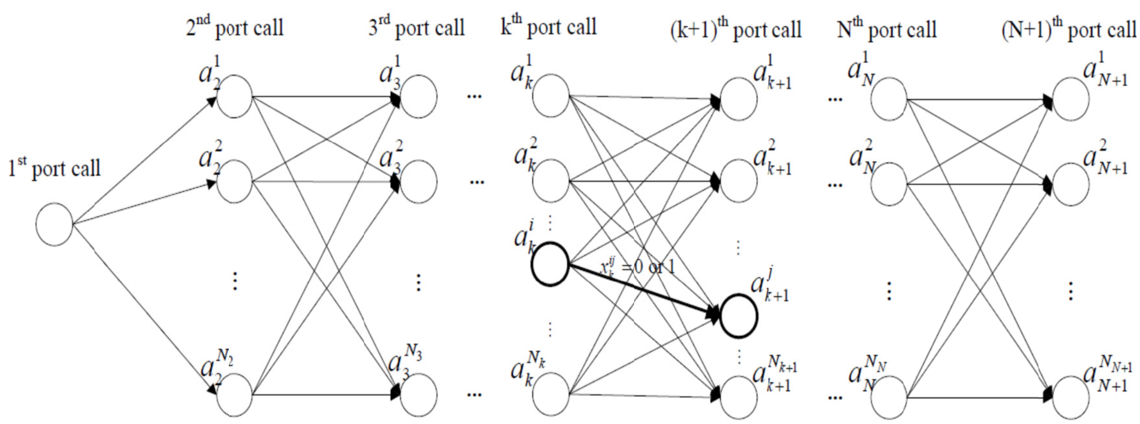

FIGURE 3 A shortest path model for the nominal fuel budget problem

Mathematically, if we define a binary variable x_k^{ij} indicating whether the link from the node for a_k^i to that for a_{k+1}^j is chosen in the shortest path, the nominal fuel budget problem can be formulated as a 0-1 integer programming model:

[NOMINAL] $$\min \quad F^{NOMINAL} = \sum_{k=1}^{N} \sum_{i=1}^{N_k} \sum_{j=1}^{N_{k+1}} f_k^{ij} \cdot \left(a_{k+1}^{j} - \left(a_k^{i} + p_k\right)\right) \cdot x_k^{ij} \tag{3}$$

subject to

$$\sum_{i=1}^{N_k} \sum_{j=1}^{N_{k+1}} x_k^{ij} = 1, \quad \forall k = 1,...,N \tag{4}$$

$$\sum_{j=1}^{N_{k-1}} x_{k-1}^{ji} = \sum_{j=1}^{N_{k+1}} x_k^{ij}, \quad \forall k = 2,3,...N, \quad \forall i = 1,...,N_k \tag{5}$$

$$x_k^{ij} \in \{0,1\}, \quad \forall k = 1,2,3,...N, \forall i = 1,...,N_k, \forall j = 1,...,N_{k+1} \tag{6}$$

where the objective function expressed by Eq. (3) calculates the total fuel consumption along a feasible path in Figure 3. Constraints (4) impose that exactly one link is chosen for each sailing leg; constraints (5) ensure the flow conservation, and constraints (6) define the binary decision variables.

ROBUST OPTIMIZATION MODEL UNDER UNCERTAINTIES

We now consider the uncertainties in ship fuel consumption rates caused by random weather conditions, especially the adverse influence of bad weather in the realistic bunker fuel budget problem. Due to the adverse influence of bad weather, the real fuel consumption rate of the ship under consideration over the link from the node for a_k^i to that for a_{k+1}^j, denoted by $\tilde{f}_k^{ij}$, is assumed to randomly change in $\left[f_k^{ij}, f_k^{ij} + \delta_k^{ij}\right]$, where f_k^{ij} is the nominal fuel consumption rate, and $\delta_k^{ij} > 0$ reflects the adverse influence of weather conditions. However, the exact probability distribution of $\tilde{f}_k^{ij}$ is genarally hard to obtain (or to pass the statistical test for common types of probability distributions). Based on the experience of ship fuel efficiency specialists in the container shipping line, the number of sailing legs on which the fuel consumption rate of this ship perturbates above its nominal value basically does not exceed Γ, among totally N sailing legs over a round voyage. $\Gamma \in \{1,2,...,N\}$ and its specific value reflects the estimation on the occurance of severe weather conditions, and thus represents the conservatism level of the ship fuel efficiency specialists in the container shipping line.

Let $\mathcal{A} = \{(k,i,j) | k = 1,...,N; i = 1,...,N_k; j = 1,...,N_{k+1}\}$ denote the set of links in Figure 3. To hedge against the worst case when the fuel consumption rates over Γ among N sailing legs randomly increase, the objective fucntion shown in Eq. (3) should be retrofited as

$$\min \quad F^{ROBUST} = \sum_{(k,i,j)\in\mathcal{A}} f_k^{ij} \left(a_{k+1}^{j} - \left(a_k^{i} + p_k\right)\right) \cdot x_k^{ij} + \max_{\{S | S\subseteq\mathcal{A}, |S|\le\Gamma\}} \sum_{(k,i,j)\in S} \delta_k^{ij} \left(a_{k+1}^{j} - \left(a_k^{i} + p_k\right)\right) \cdot x_k^{ij} \tag{7}$$

To simplify the mathematical expression, we introduce:

$$g_k^{ij} = f_k^{ij} \left(a_{k+1}^{j} - \left(a_k^{i} + p_k\right)\right), \quad \Delta_k^{ij} = \delta_k^{ij} \left(a_{k+1}^{j} - \left(a_k^{i} + p_k\right)\right), \quad (k,i,j) \in \mathcal{A} \tag{8}$$

The robust optimization model under uncertainties can be formulated as below:

[ROBUST1] $$\min \quad F^{ROBUST} = \sum_{(k,i,j)\in\mathcal{A}} g_k^{ij} \cdot x_k^{ij} + \max_{\{S | S\subseteq\mathcal{A}, |S|\le\Gamma\}} \sum_{(k,i,j)\in S} \Delta_k^{ij} \cdot x_k^{ij} \tag{9}$$

subject to constraints (4)-(6).

The second term of objective function (9) with the "max" operator is equivalent to a linear programming problem:

$$\max \sum_{(k,i,j)\in\mathcal{A}} \Delta_k^{ij} \cdot x_k^{ij} \cdot y_k^{ij} \tag{10}$$

subject to

$$0 \le y_k^{ij} \le 1, \quad (k,i,j)\in\mathcal{A} \tag{11}$$

$$\sum_{(k,i,j)\in\mathcal{A}} y_k^{ij} \le \Gamma \tag{12}$$

Let $\mu_k^{ij},(k,i,j)\in\mathcal{A}$ and λ be the dual variables with respect to of constraints (11) and (12), respectively. Solving the linear programming model (10) - (12) is equivalent to solving its dual program:

$$\min \quad \Gamma\cdot\lambda + \sum_{(k,i,j)\in\mathcal{A}} \mu_k^{ij} \tag{13}$$

subject to

$$\mu_k^{ij} + \lambda \ge \Delta_k^{ij} \cdot x_k^{ij}, \quad (k,i,j)\in\mathcal{A} \tag{14}$$

$$\lambda, \mu_k^{ij} \ge 0, \quad (k,i,j)\in\mathcal{A} \tag{15}$$

Model [ROBUST1] can thus be rewritten as follows:

[ROBUST2] $$\min \quad F^{ROBUST} = \sum_{(k,i,j)\in\mathcal{A}} g_k^{ij}\cdot x_k^{ij} + \Gamma\cdot\lambda + \sum_{(k,i,j)\in\mathcal{A}} \mu_k^{ij} \tag{16}$$

subject to constraints (4)-(6), (14) and (15).

Compared to model [ROBUST1], model [ROBUST2] has more decision variables. However, model [ROBUST2] becomes a mixed-integer linear programming (MILP) model which could be solved by a number of optimization solvers such as CPLEX and Gurobi. In fact, we can do better to solve the robust model. As a component of the robust optimization theory, Bertsimas and Sim (*24*) prove that the robust counterpart of a polynomially solvable combinatorial optimization problem is also polynomially solvable and propose the solution algorithm. We apply their theoretical findings and solution algorithm to model [ROBUST2], and describe them in next section.

SOLUTION METHOD

We rearrange the link index set $\mathcal{A}$ as $\mathcal{O}$ in the decreasing order of $\Delta_k^{ij},(k,i,j)\in\mathcal{A}$, namely,

$$\Delta_1 \ge \Delta_2 \ge \ldots \ge \Delta_{|\mathcal{O}|} \tag{17}$$

where $|\mathcal{O}|=|\mathcal{A}|$. Based on this new index set, $g_k^{ij}, x_k^{ij}, (k,i,j)\in\mathcal{A}$ are replaced by g_o and $x_o, o\in\mathcal{O}$ respectively. We define $\Delta_{|\mathcal{O}|+1}=0$. The closely-related theoretical findings of Bertsimas and Sim (*24*) can be re-expressed by the following theorem, for our specific model [ROBUST2].

Theorem 1. Model [ROBUST2] can be optimally solved by solving totally $|\mathcal{O}|+1$ nominal shortest path problems:

$$F^{ROBUST} = \min_{l=1,\ldots,|\mathcal{O}|+1} G^l \tag{18}$$

where for a specific l, the problem G^l is defined as

$$G^l = \Gamma \cdot \Delta_l + \min\left[\sum_{o=1}^{|\mathcal{O}|} g_o \cdot x_o + \sum_{o=1}^{l} \left(\Delta_o - \Delta_l\right) \cdot x_o\right] \quad (19)$$

in which the first term is a constant, and the second term is a nominal shortest path problem.
Proof. Follow the same process of Bertsimas and Sim (*24*), which first eliminates the dual variables $\mu_k^{ij}, (k,i,j) \in \mathcal{A}$ based on the structural property of optimal solutions, and then λ by employing the fact that $x_k^{ij}, (k,i,j) \in \mathcal{A}$ are binary decision variables. □

Remarks for Theorem 1: (a) compared to the shortest path problem shown in Figure 3, the problem G^l increases the cost (bunker fuel consumption) over link $o \in \{1,...,l\}$ to $g_o + (\Delta_o - \Delta_l)$ while it leaves the cost over other links unchanged; (b) the shortest path problem in the second term of G^l is independent of the specific value of Γ, which supports the computational merit that it only requires solving a set of shortest path problems $\{G^l\}_{l=1}^{|\mathcal{O}|+1}$ once when the robust fuel consumption values at different levels of conservatism of industrial fuel efficiency specialists are needed no matter how many possible values of Γ are chosen; (c) if $\Delta_l = \Delta_{l+1}$, the two optimization problems of G^l and G^{l+1} will be the same, which provides an additional computational advantage that the times for solving shortest path problems can be reduced to the total number of different nonzero Δ_l plus 1; and (d) a dummy terminal node can be added into the shortest path problem involved in G^l to facilitate using the Dijkstra's algorithm, although the framework proposed by Bertsimas and Sim (*24*), and thus the derivation process to robust optimization models [ROBUST] and [ROBUST2], do not support using the dummy terminal node and the dummy links to it.
Based on Theorem 1 and the algorithm of Bertsimas and Sim (*24*) for a general combinatorial optimization problem, the solution algorithm for our ship fuel budget robust optimization model can be designed as follows:

Solution Algorithm

Step 1. Sort the indexes/arcs (k,i,j) in $\mathcal{A}$ in the decreasing order of its fuel consumption deviation $\Delta_k^{ij} = \delta_k^{ij}\left(a_{k+1}^j - \left(a_k^i + p_k\right)\right)$ and obtain a new index array $\mathcal{O}$:

$$\Delta_1 \geq \Delta_2 \geq ... \geq \Delta_{|\mathcal{O}|}$$

Step 2. For $l = 1,2,...,|\mathcal{O}|+1$, solve the shortest path problem G^l represented by (19);

Step 3. Find $l^* = \arg\min_{l=1,...,|\mathcal{O}|+1} G^l$, and let the optimal bunker fuel budget value of the ship over a round voyage be G^{l^*} and the robust ship schedule as the shortest path suggested by G^{l^*}.

Let us analyze the computational time complexity of the above solution algorithm. The time complexity of sorting in Step 1 is $O\left(|\mathcal{A}|\log\left(|\mathcal{A}|\right)\right)$; Step 2 solves shortest path problems with say the Dijkstra's algorithm $|\mathcal{O}|+1 = |\mathcal{A}|+1$ times, and thus needs computational time of complexity $O\left(|\mathcal{A}|\sum_{k=1}^{N+1} N_k\right)$; Step 3 finds the minimum among $|\mathcal{O}|+1=|\mathcal{A}|+1$ values and thus consumes computational time of complexity $O(|\mathcal{A}|)$. Consequently, the proposed solution algorithm is a polynomial time method.

CASE STUDY

We use the Asia-Europe service LP4 operated by APL in this case study, and the ship under consideration is assumed to be ship S1 shown in Figures 1 and 2. The port rotation, port durations and arrival time windows are tabulated in Table 1. Each arrival time window is discretized on an hourly basis, which is a fine time-resolution for a long shipping voyage such as an Asia-Europe service generally lasting for more than two months. For the influence of different discretization granularities on solution optimality, the interested readers are referred to the work of Fagerholt et al. (*13*). The regression curve in Figure 1 and the curve representing a wave height of 7 m in Figure 2 are utilized to define the lower and upper bound of $\left[f_k^{ij}, f_k^{ij}+\delta_k^{ij}\right]$ in which the fuel consumption rates of S1 perturbate.

Computational Performance

Model [ROBUST2] is a mixed-integer linear programming problem which might be optimally solved by commercial optimization solvers such as CPLEX and Gurobi. To compare the computational performance of the Branch and Cut (B&C) algorithm and that of the solution algorithm presented above, we solve model [ROBUST2] with both IBM ILOG CPLEX 12.6 and the proposed solution algorithm, in which process YALMIP (*26*) is used to formulate [ROBUST2] in MATLAB. The time limit for the B&C algorithm in CPLEX is set to 300 seconds in view of the efficiency of the proposed solution algorithm.

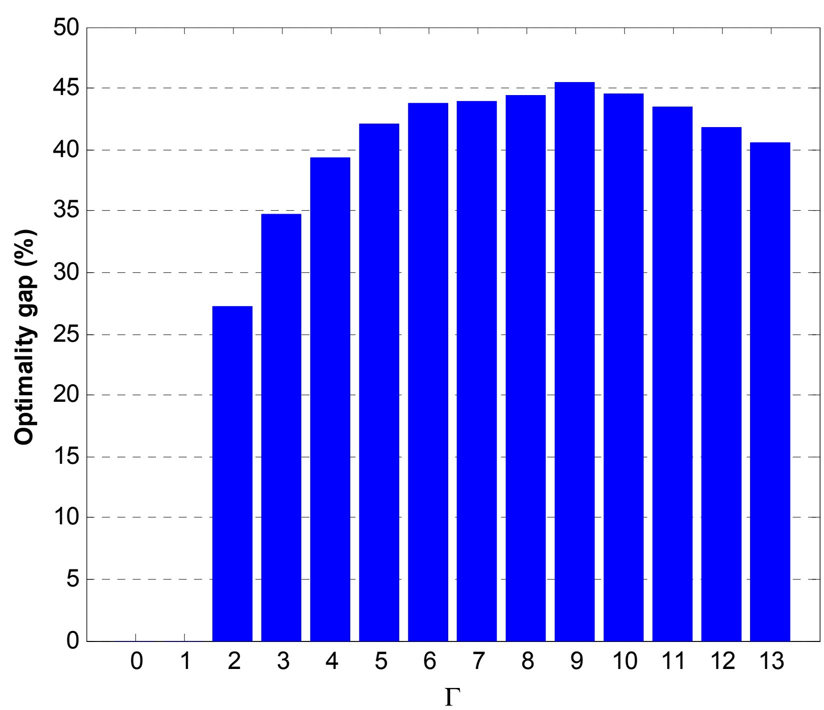

FIGURE 4 Optimality gaps when CPLEX terminates at the 300-s time limit

The whole network totally has 1+24×12+16=305 nodes and 5875 arcs over which there are totally 470 different deviation values of fuel consumption (Δ_l). The B&C algorithm in CPLEX can solve the nominal model [NOMINAL], i.e. $\Gamma = 0$, in less than 1 second. This can be easily understood from the theoretical viewpoint because model [NOMINAL] is a shortest path problem and it possesses the structural property of *totally unimodularity*. However, when $\Gamma \geq 1$, model [ROBUST2] seems much harder to solve and CPLEX cannot solve model [ROBUST2] to optimality within 300 seconds except for $\Gamma = 1$. The optimality gaps with different values of Γ are depicted in Figure 4. This is partly because model [ROBUST2] loses the nice property of totally unimodularity and much more dual variables and relevant constraints enter the model.

The proposed solution algorithm needs to solve 470+1=471 shortest path problems. It can solve model [ROBUST2] over this test case to optimality in 15 seconds according to our experiments, which fully demonstrates its high computational efficiency compared to commercial solvers and strongly underpins its industrial application in decision support systems.

Robust Shipping Schedules and Price of Robustness

The robust shipping schedules worked out by the proposed solution algorithm when $\Gamma \in \{1, 2, ..., 6\}$ are shown in Table 2. We do not list the results when $\Gamma \geq 7$ because the probability of a ship experiencing 7-meter bow waves over more than 7 among 13 legs is too low in practice. Meanwhile, we plot the robust objective values, i.e. F^{ROBUST} (fuel consumption over a round voyage under uncertainties), and the nominal objective values of these robust shipping schedules, i.e. $F^{NOMINAL}$ of the robust schedules, in Figure 5.

TABLE 2 Shipping schedules under different robustness protection levels (Γ)

Γ	Shipping schedule												
	YAN	YAT	SIN	SUZ	KLV	SOU	HF8	RTM	SUZ	JED	SIN	YAT	NTB
0	5	88	193	533	744	768	833	899	1183	1249	1584	1746	1816
1	5	88	193	552	744	768	833	899	1183	1249	1584	1746	1816
2	5	88	193	552	744	768	833	899	1183	1249	1584	1746	1816
3	5	88	193	533	744	768	833	899	1183	1249	1584	1746	1816
4	5	88	193	533	744	767	827	890	1191	1249	1584	1746	1816
5	4	80	193	533	744	767	827	890	1191	1249	1584	1746	1816
6	4	80	193	533	744	767	827	890	1191	1249	1584	1752	1816

Note: unit: hour; departure time from NTB (first port call) is considered as time zero.

It can be seen that with the increase of the value of Γ, model [ROBUST2] pays more and more attention to the robust part (the second and third terms) of the objective function expressed by Eq. (16) to hedge against increased anticipated uncertainties, which causes the total objective values to increase dramatically. In other words, when the levels of conservatism of industrial specialists are lifted, the robustness of the shipping schedule is improved to hedge against the perturbation of ship fuel consumption rates due to severe weather conditions, but we need to pay more to the ship fuel budget and sacrifice the nominal optimality. Ship fuel efficiency specialists in a shipping line can choose a suitable value of Γ based on their risk preference level.

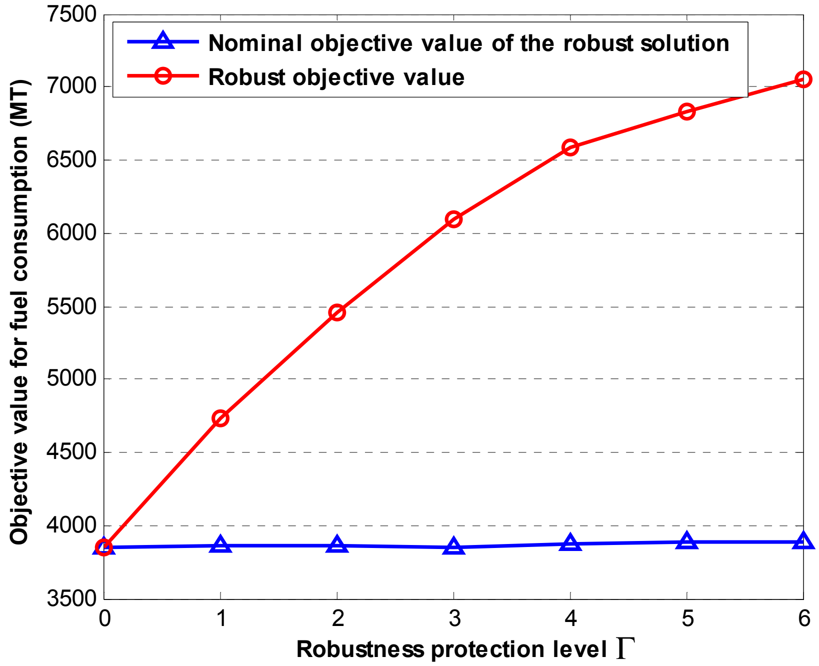

FIGURE 5 Fuel budget values of ship S1 over a round voyage at different robustness protection levels

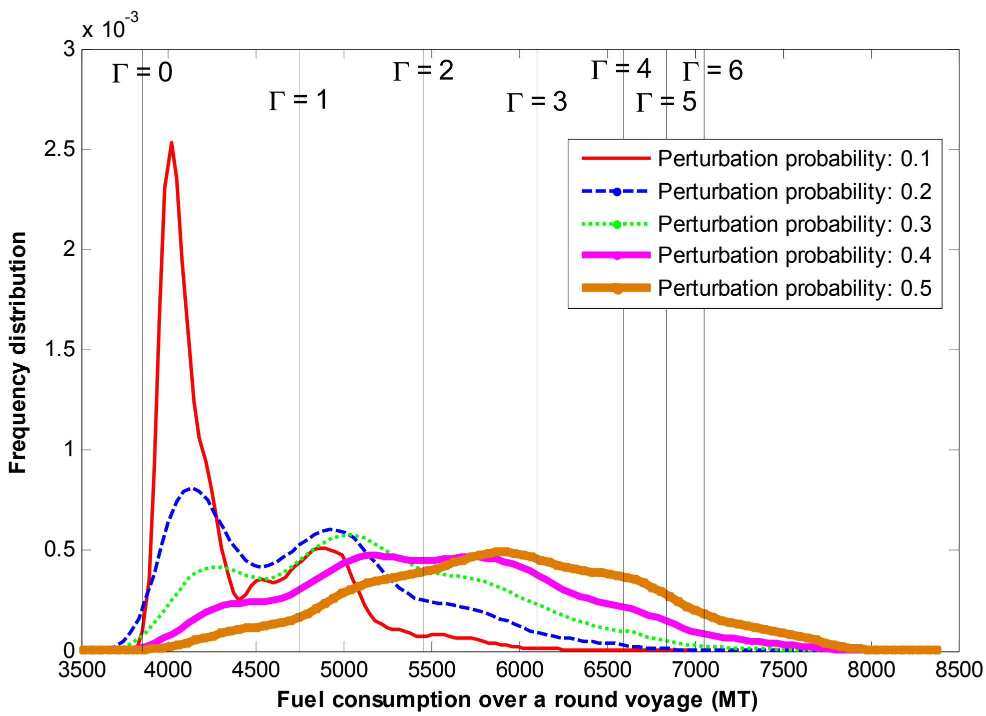

FIGURE 6 Distributions of fuel consumption of ship S1 over a round voyage with different perturbation probabilities of bunker consumption

Simulation Results

To validate whether the proposed robust optimization model can produce good fuel budget values in real shipping situation, we randomly generate 100 feasible shipping schedules of service LP4, and evaluate the fuel consumption implied by these schedules under uncertain weather conditions. To simulate the influence of severe weather, we assume that the actual fuel consumption rate of ship S1 over each network link independently perturbs, with probability $\alpha \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, from its nominal value f_k^{ij} to $f_k^{ij} + \delta_k^{ij}$. For each value of α, we generate 100 random scenarios and calculate the fuel consumption of ship S1 for each feasible schedule over each random scenario (totally we have 100×100=10000 schedule-scenario combinations for each value of perturbation probability α) . The distributions of fuel consumption of S1, together with the fuel budget values produced by our robust optimization models, are ploted as the curves/lines shown in Figure 6.

It can be seen that when the perturbation probability $\alpha \le 0.2$, the robust objective value with $\Gamma = 2$ will be a good budget value for bunker fuel consumption. Similarly, with $\Gamma = 4, 6, 6$, the proposed robust model could produce good budget values if the perturvation probability α caused by severe weather conditions is 0.3, 0.4 and 0.5, respectively. Figure 6 also indicates the possibility that actual fuel consumption is higher than these budget values. This is implicated with the fact that these feasible schedules (tested in experiments and adopted in practice) are not necessarily optimal from the viewpoint of fuel consumption management. We thus can see the importance of both "robustness analysis" and "optimal schedule design", which is the spirit of robust optimization theory.

CONCLUSIONS

This paper has dealt with the fuel budget problem for a container ship over a single round voyage, inspired by the liner shipping industrial trend in implementing ship fuel efficiency management programs. This study takes an initiative to examine this management issue with practical significance in liner shipping studies. To address the adverse influence of the perturbation of ship fuel consumption rates under severe weather conditions on bunker fuel budget estimation, we employ the state-of-the-art robust optimization techniques developed by Bertsimas and Sim (*24*) and build a robust optimization model for the fuel budget problem. Although the robust optimization model can be transformed to a MILP model with the possibility to be solved by commercial solvers, we utilize the algorithmic findings on a general combinatorial problem by Bertsimas and Sim (*24*) and design a polynomial time algorithm based on solutions of multiple shortest-path problems. A case study of the LP4 service operated by APL demonstrates the computational competence of the proposed algorithm and shows that the proposed model can work out good fuel budget values at different levels of conservatism under realistic but uncertain situations.

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